Spectro-Perfectionism: An Algorithmic Framework for Photon Noise-Limited Extraction of Optical Fiber Spectroscopy

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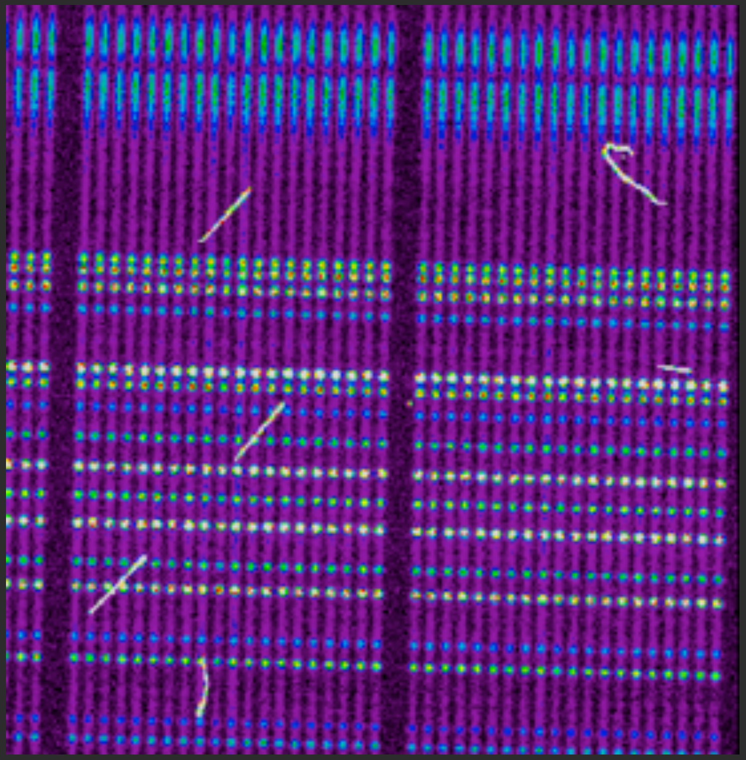
Exoplanet PRVs - PSU - 2010 Aug 19

Beware of...

- What you think you know about LSFs and cross-sectional profiles
- Extragalactic astronomers proffering advice
- Fake data

Spectro-Perfectionism:

What is the *right* way to go from this:



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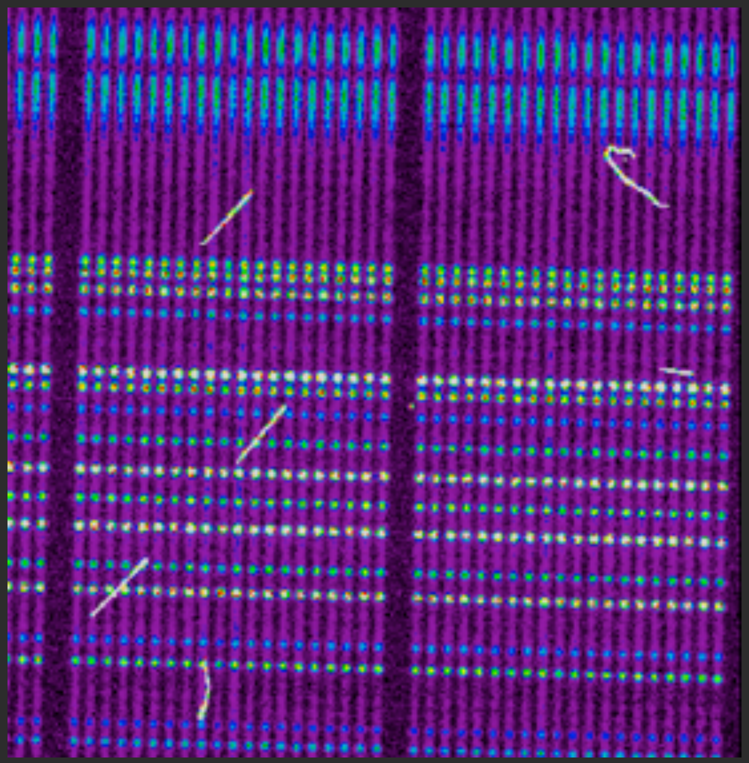
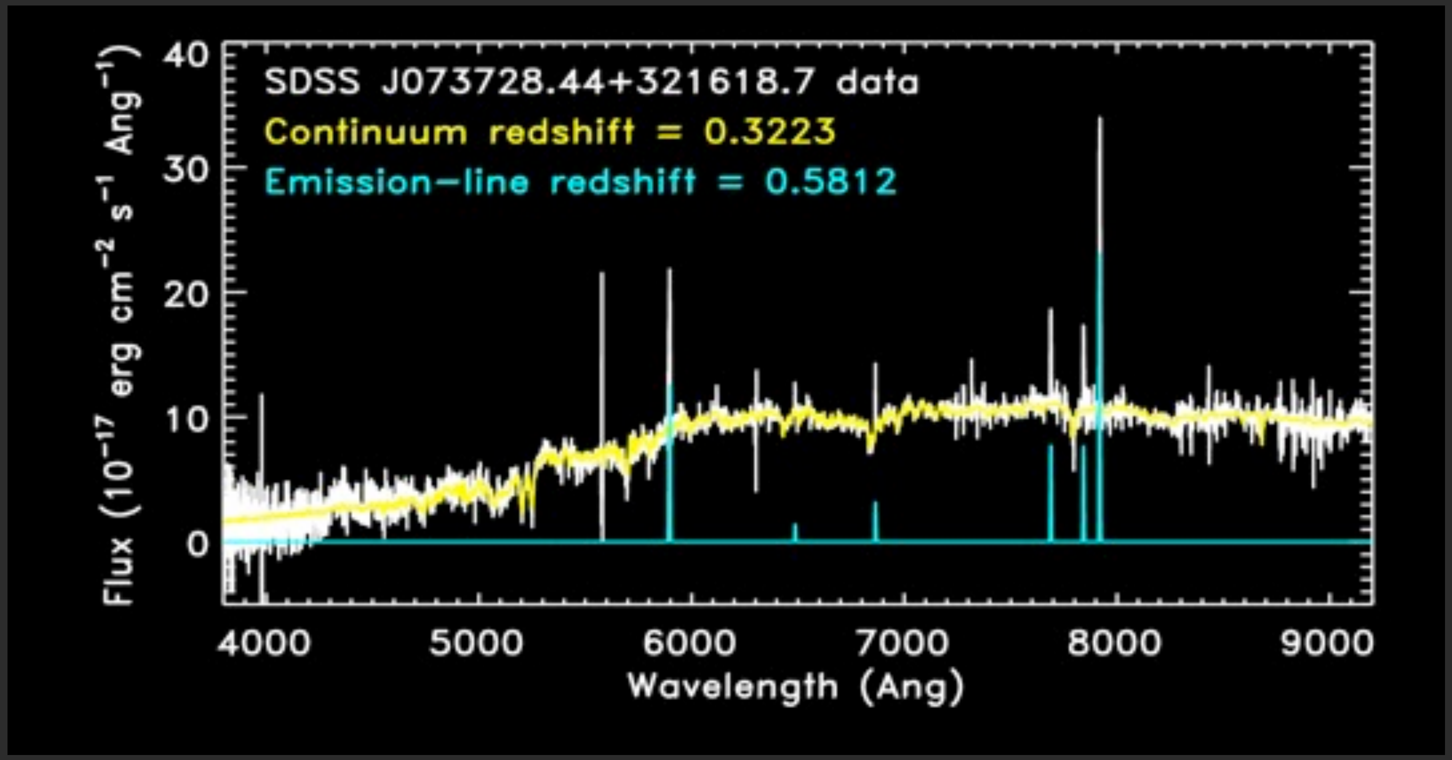


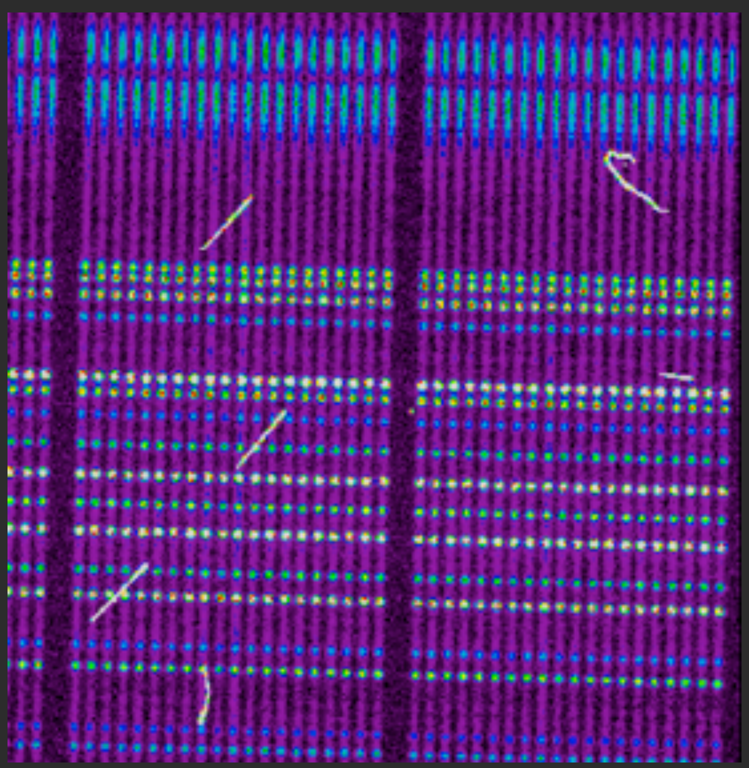
... to this:



?

Spectro-Perfectionism:

Bolton & Schlegel (2010, PASP, 122, 248)



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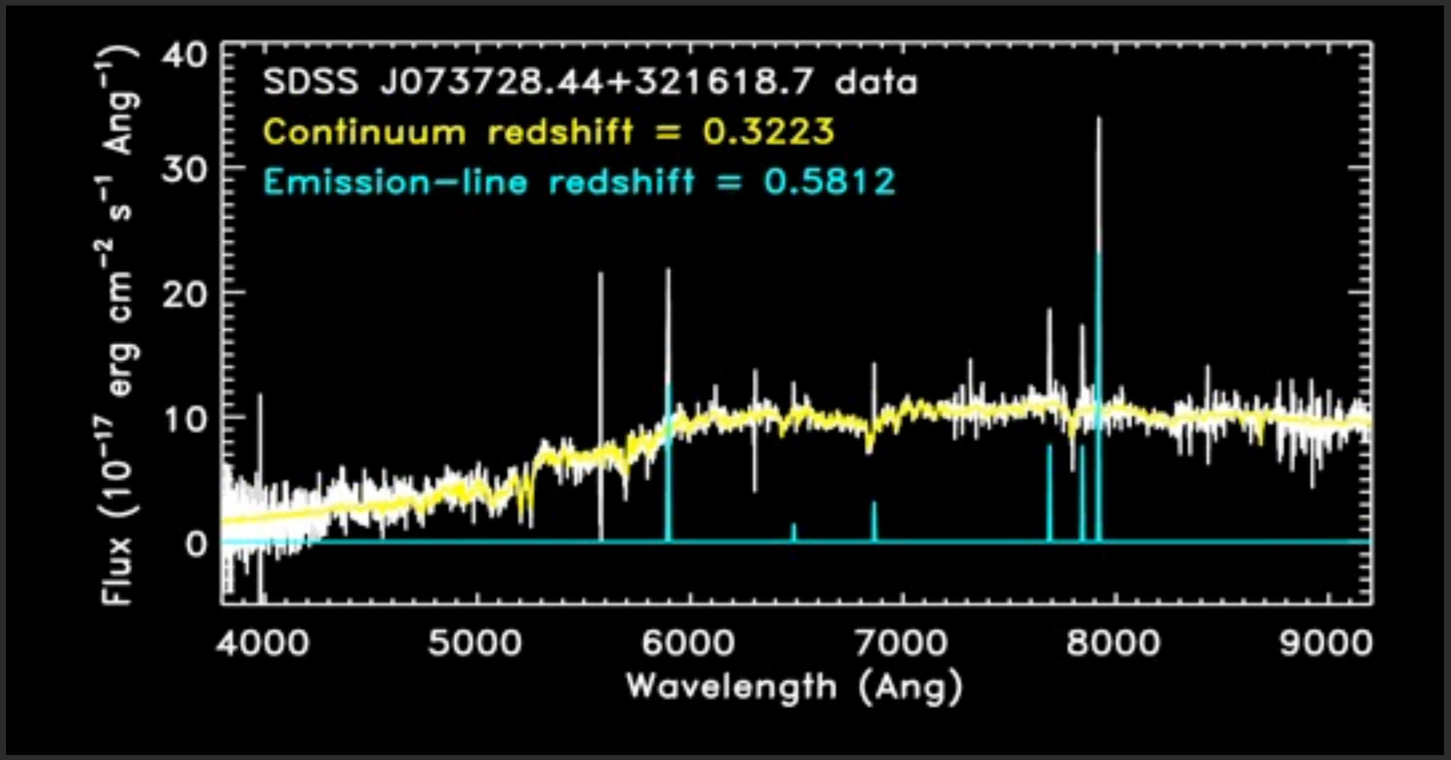


Hasn't this problem been solved?

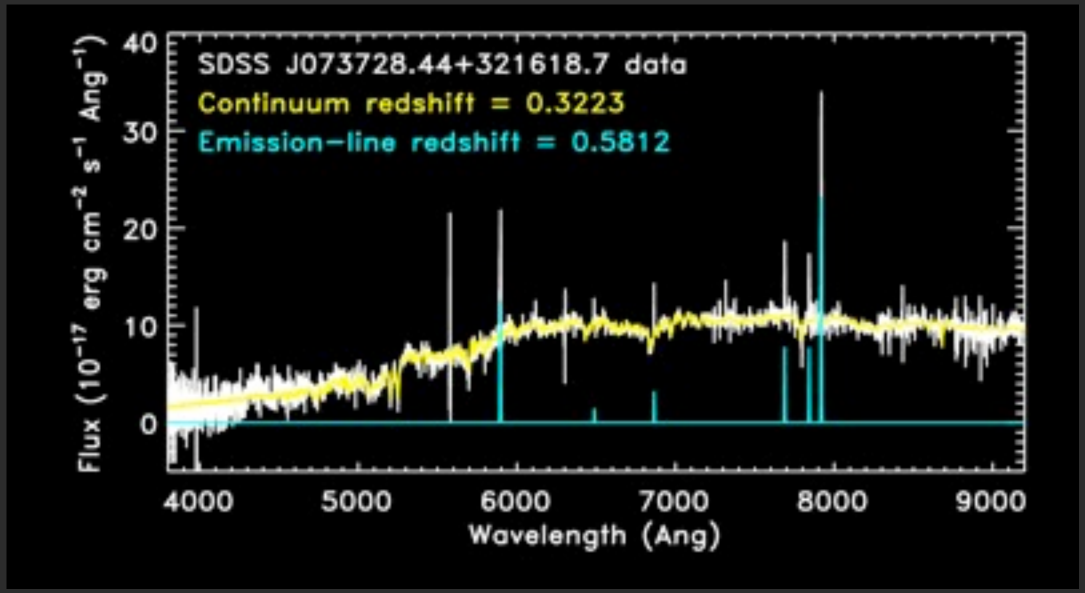
Yes, sort of...

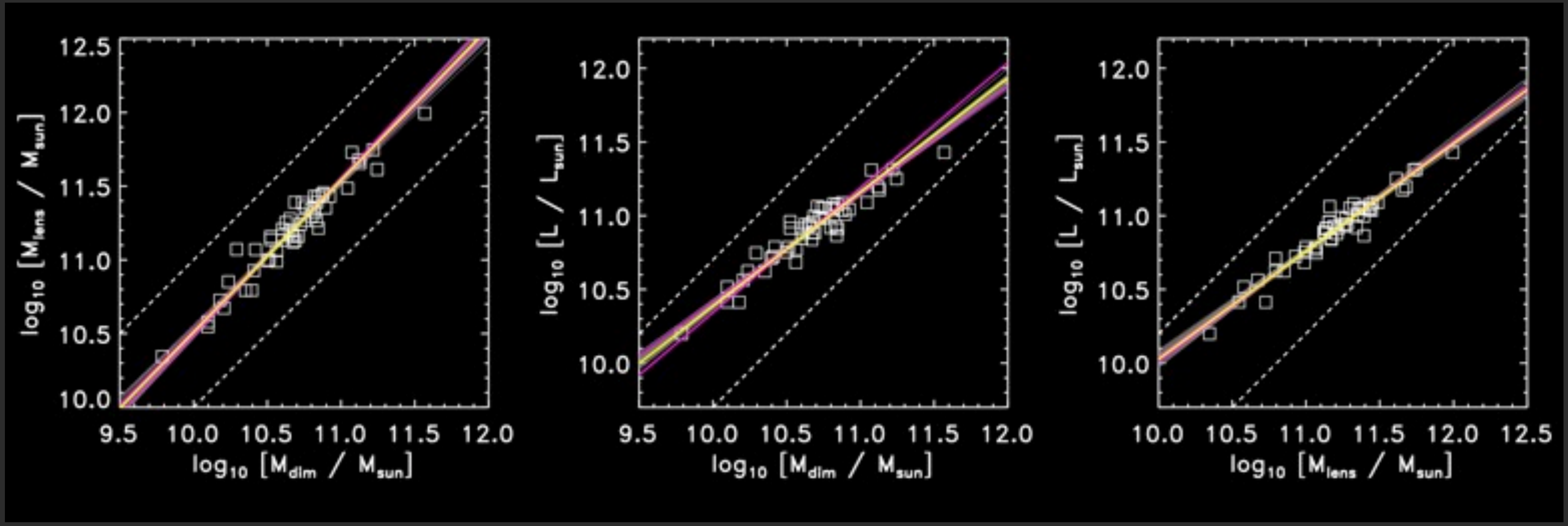
Hasn't this problem been solved?

"Perfectionism is a disease"

-PLS

Why do I care?





www.SLACS.org

Why might you care?

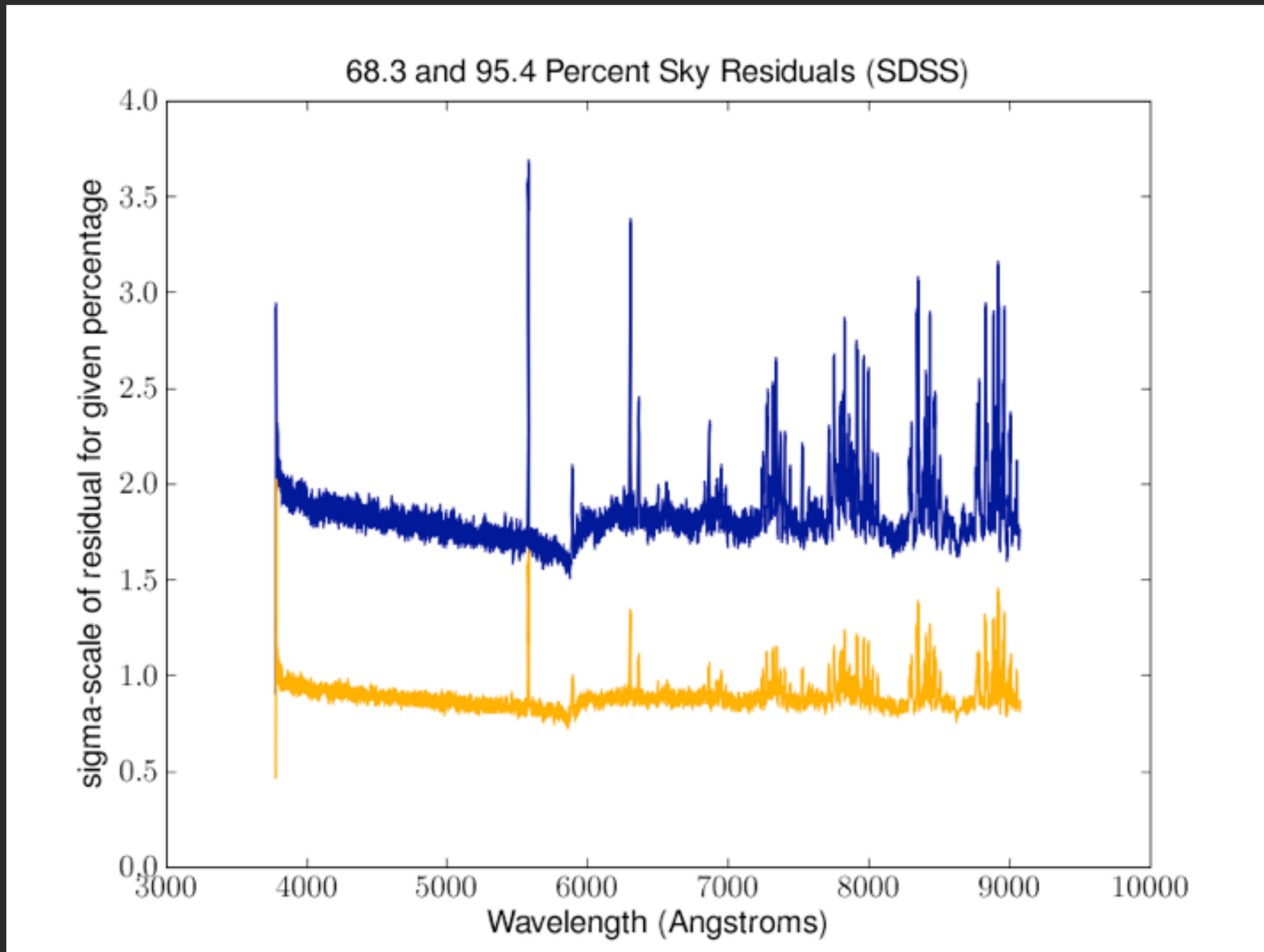
You're already forward-modeling your spectra. Why not forward-model your raw data, too?

Signal-to-noise regimes:
- SNR ~ 100: systematics limited
- SNR ~ 10: statistics limited
- SNR ~ 1: systematics limited

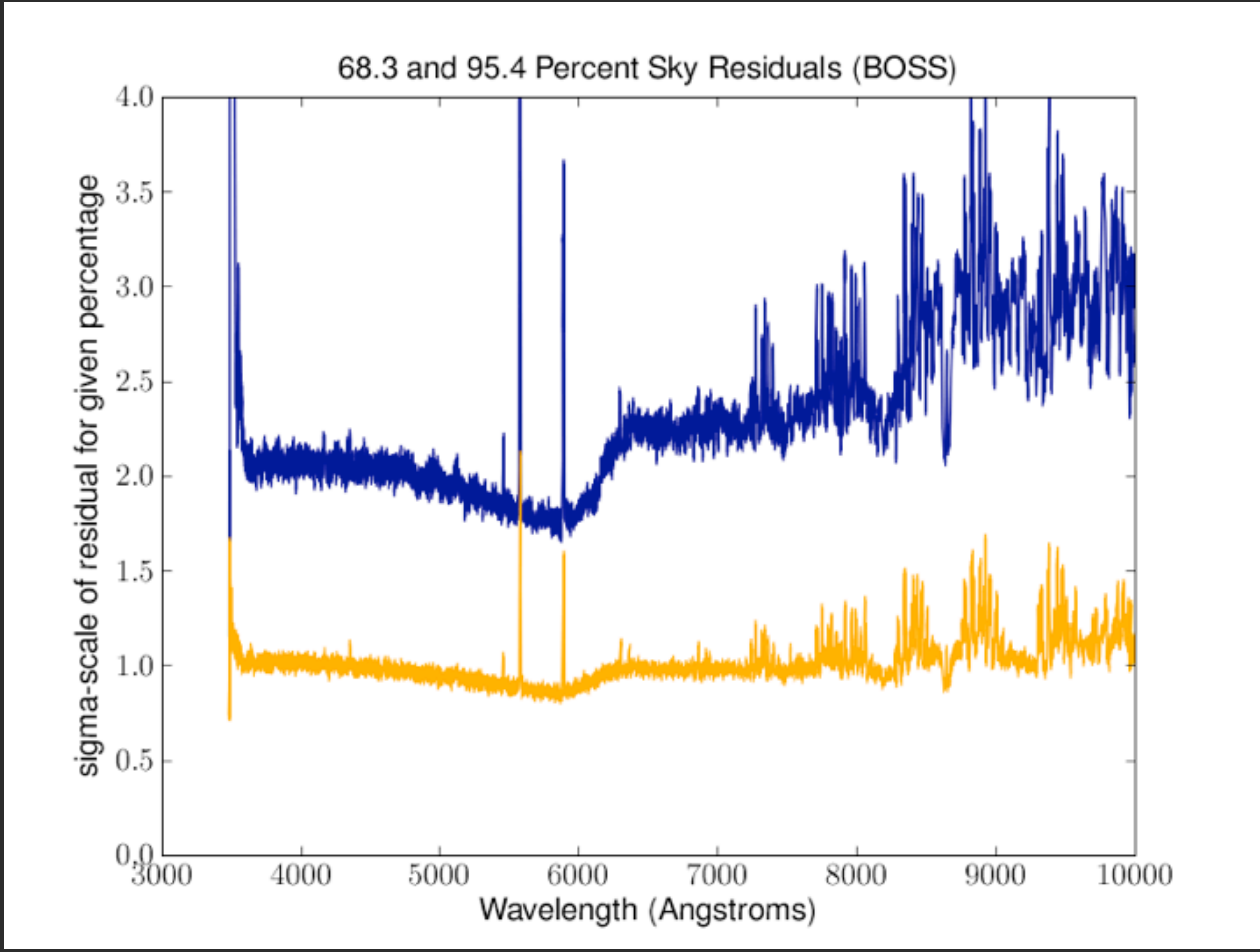
Astronomy as experimental physics: we don't control the accelerator, so best to control and understand the detector well!

Spectra get fainter; sky stays as bright as ever.

Systematics of sky subtraction



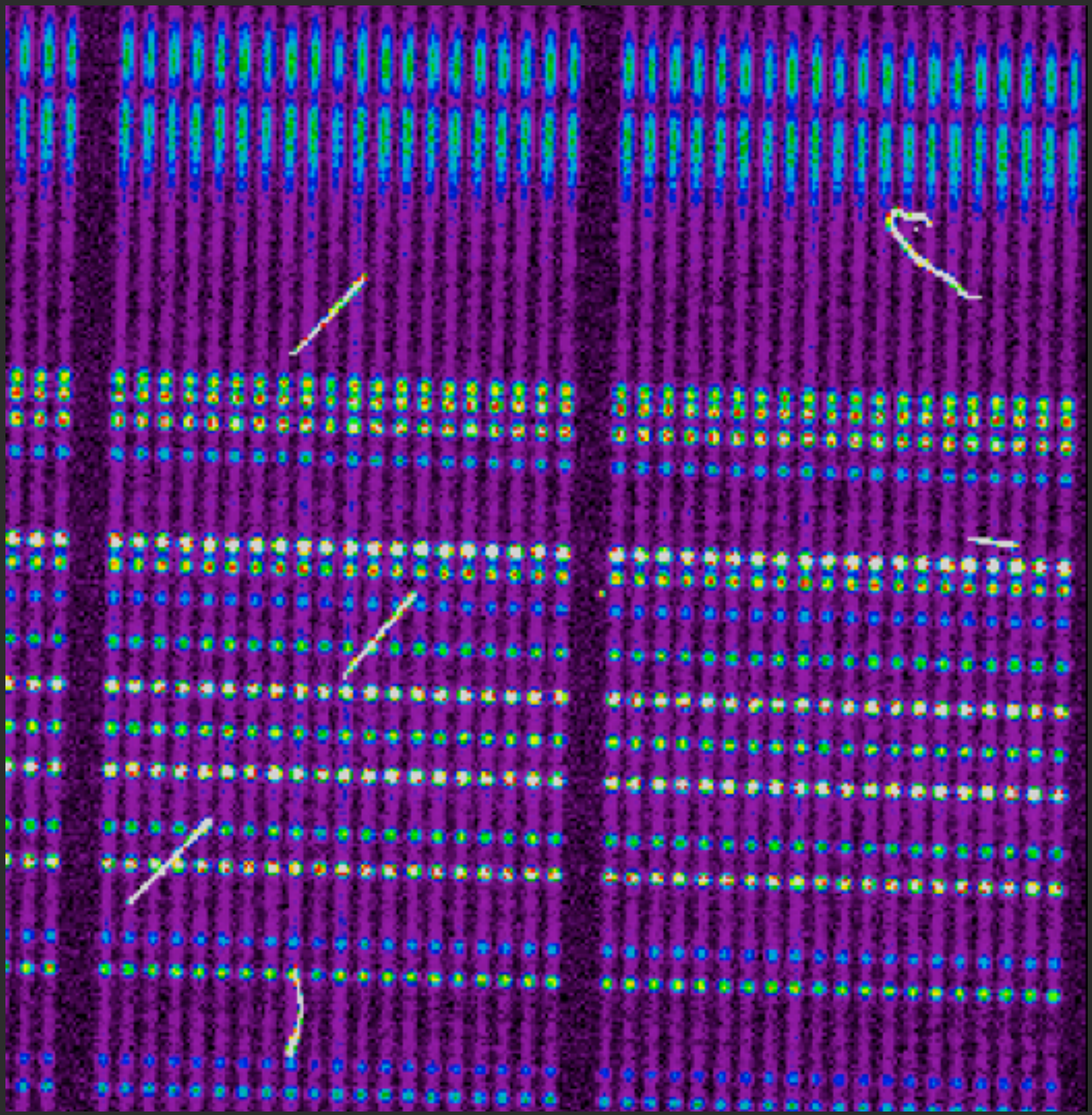
Systematics of sky subtraction



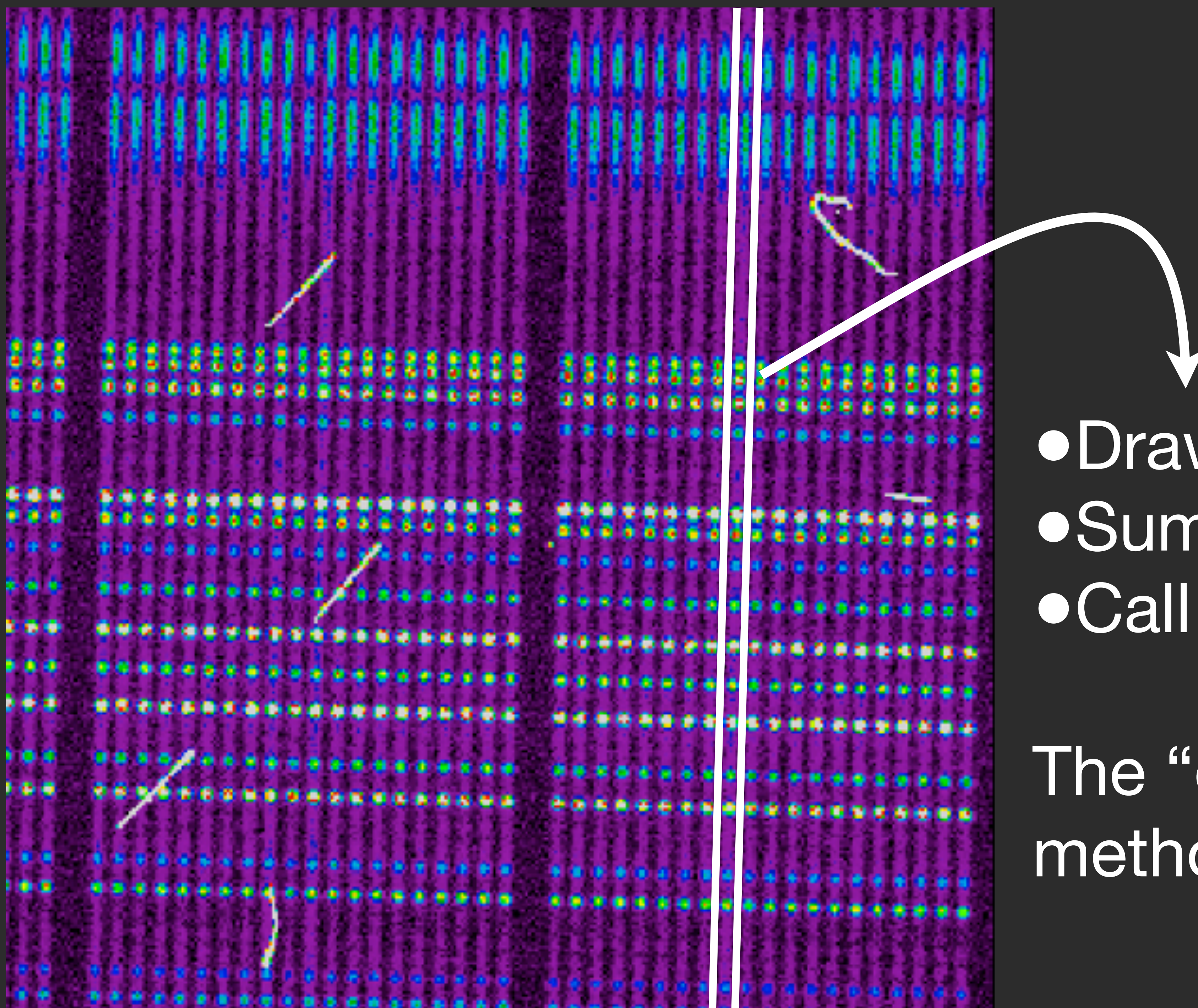
What do we want in an extraction?

- Define in terms of objective scalar optimization
- Generate noise-limited model of all 2D frames
- Allow optimal weighting
- Do not degrade resolution
- Characterize resolution accurately
- Avoid correlations in extracted 1D samples
- Propagate errors correctly (for correct chi^2)
- Preserve these properties in multi-frame coadds
- Allow foreground estimation and subtraction in the presence of optical non-uniformities
- Deliver something that fits an astronomer's understanding of "the extracted spectrum"
- Make it easy to implement

Boxcar extraction



Boxcar extraction



- Draw two lines
- Sum enclosed counts
- Call that your spectrum

The “quick and dirty” method.

Boxcar scorecard

- ☐ Define in terms of objective scalar optimization
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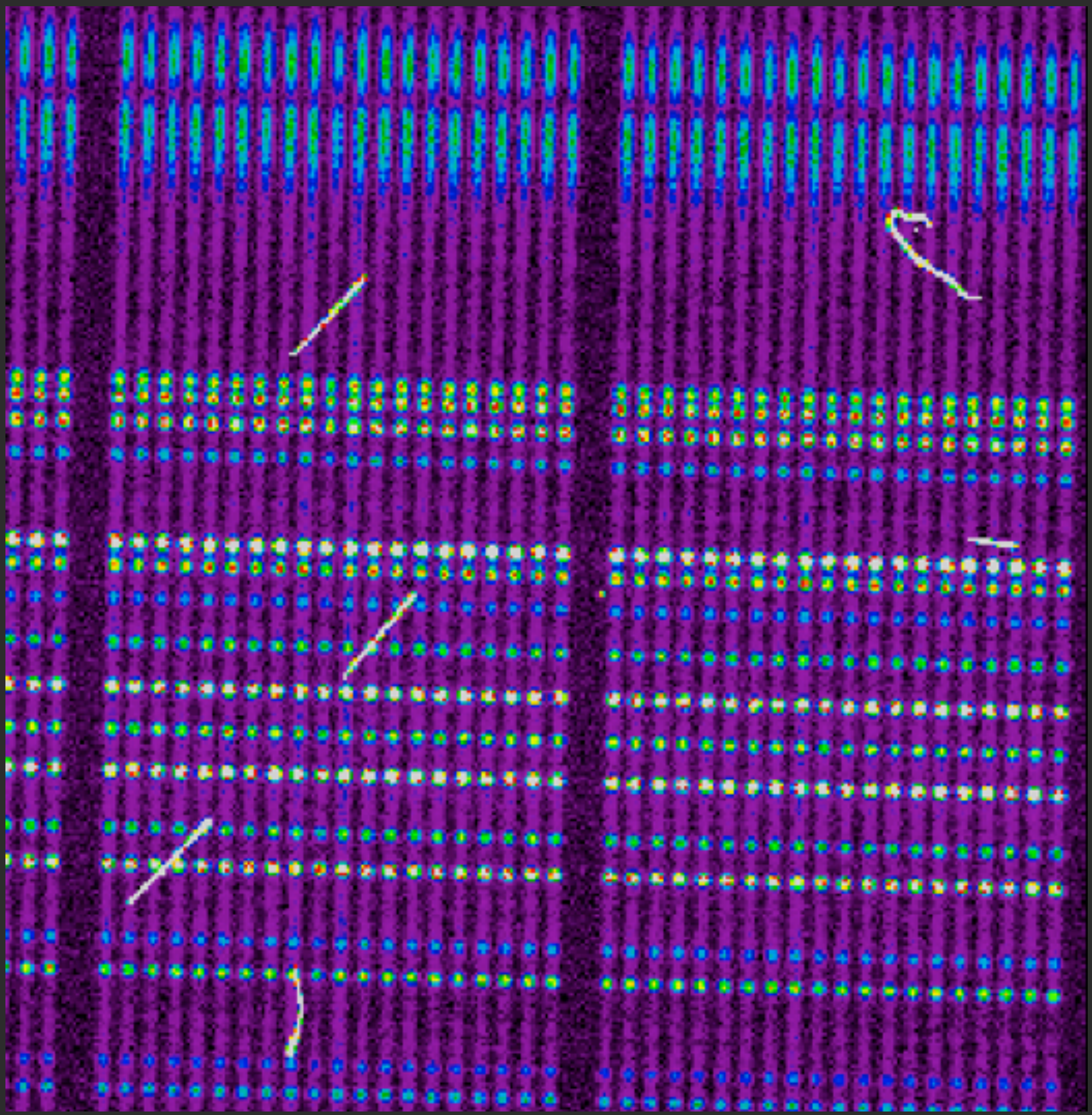
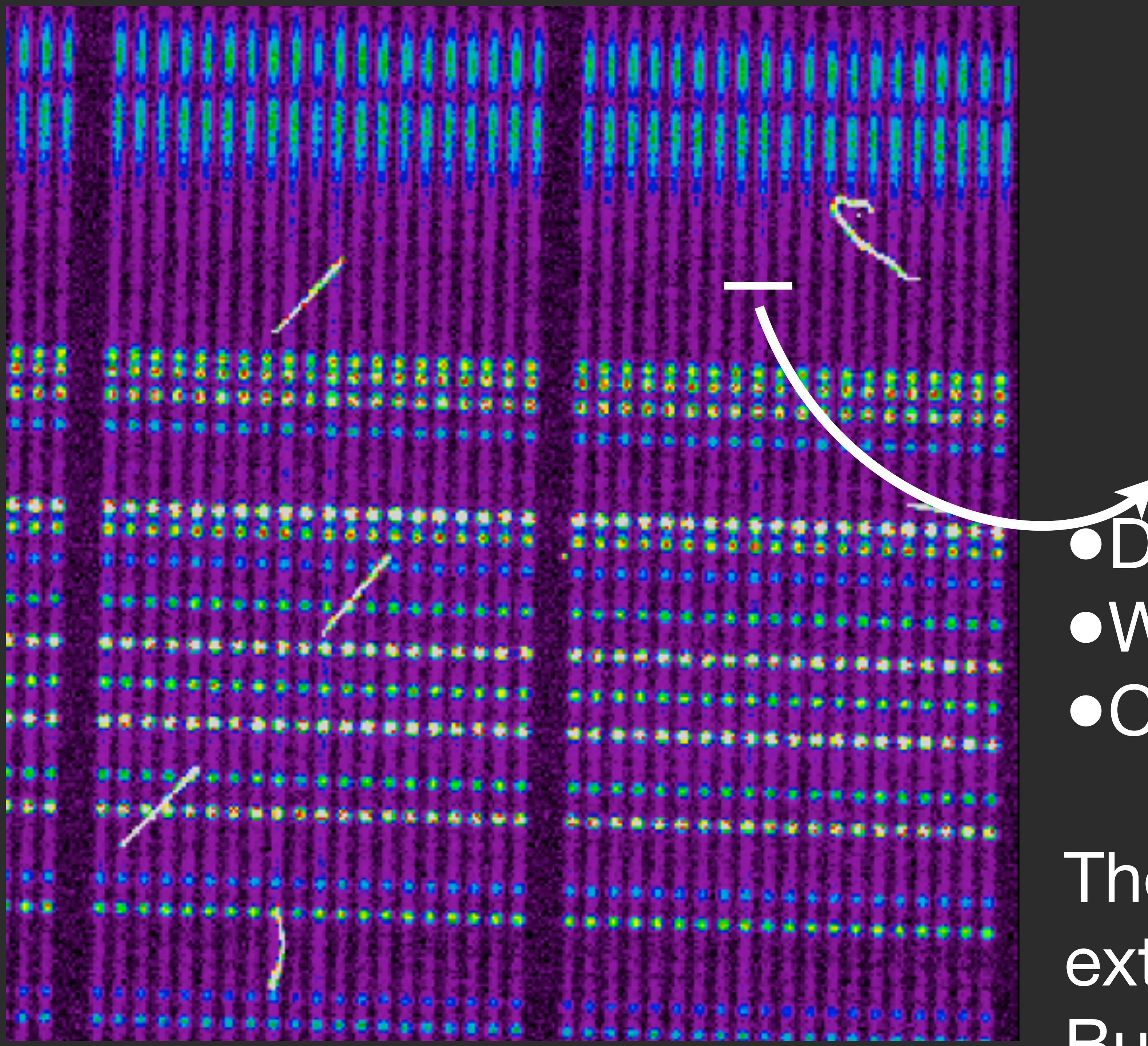
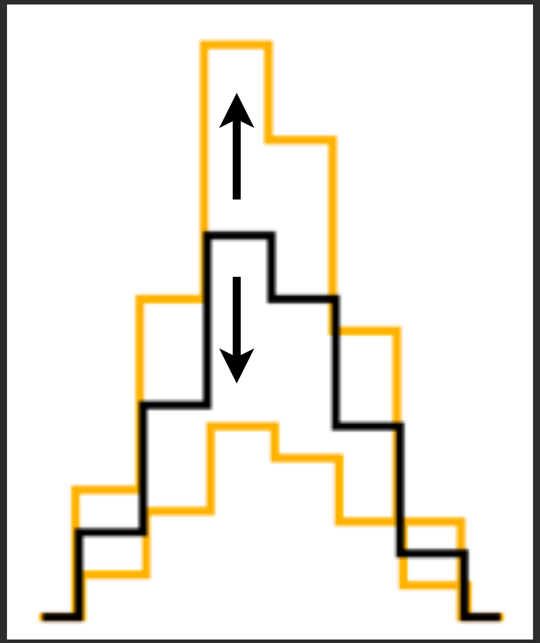
Optimal extraction



Optimal extraction

Hewett et al. 1985;
Horne 1986





- Determine cross-sec'n
- Weighted amplitude fit
- Call that your spectrum

The current standard in extraction (e.g., SDSS: Burles & Schlegel)

Optimal-extraction scorecard

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(almost)

The general problem

$$A_{jk} (f_k + s_k) = p_j + n_j + b_j$$

A_{jk}: Calibration matrix

f_k: Input flux vector

s_k: Input background vector

p_j: Pixel count (data) vector

n_j: Pixel noise vector

b_j: Internal background vector

The general problem

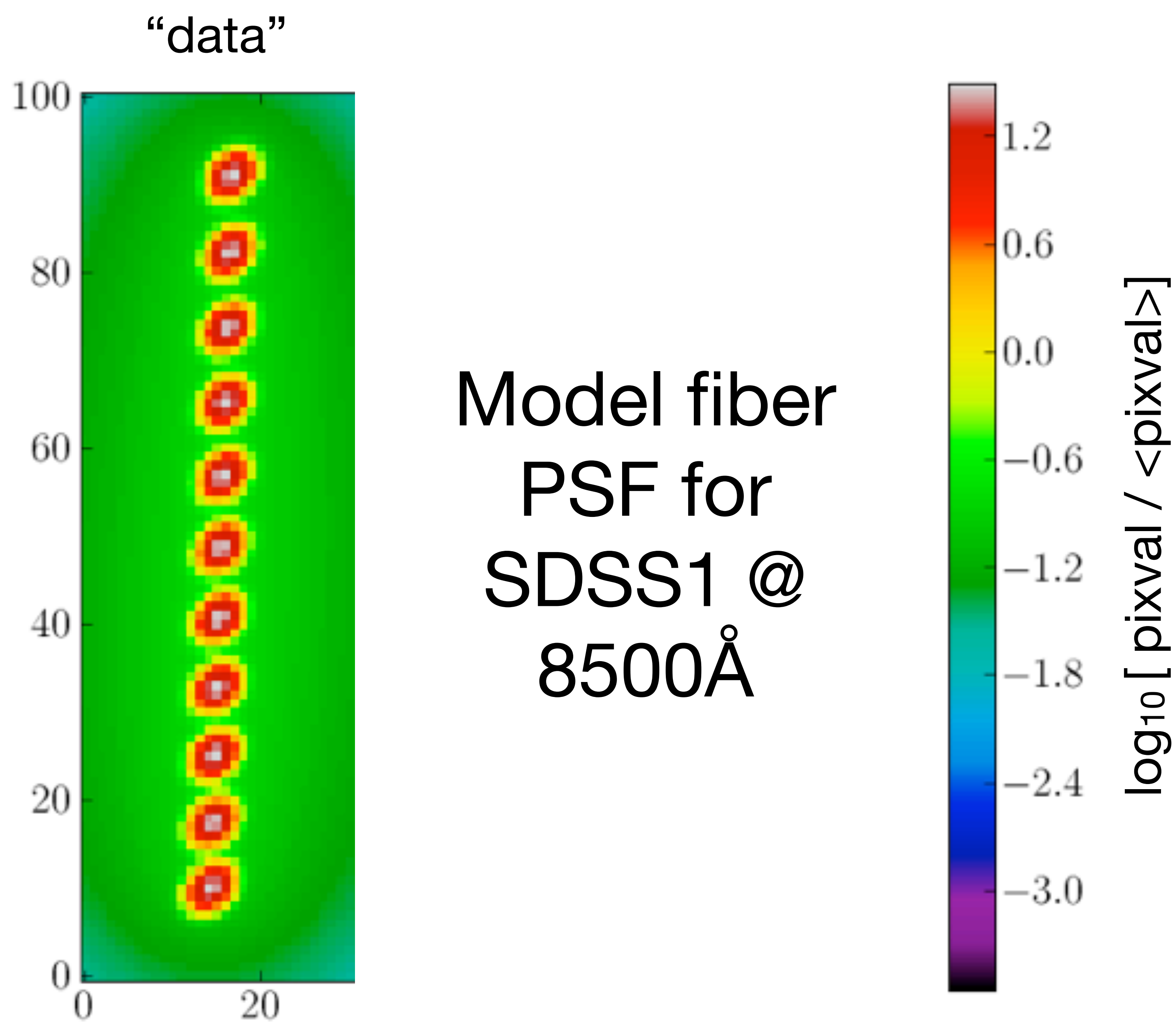
$$A_{jk} (f_k + s_k) = p_j + n_j + b_j$$

A_{jk}: Calibration matrix

A sparse matrix that unifies and extends:

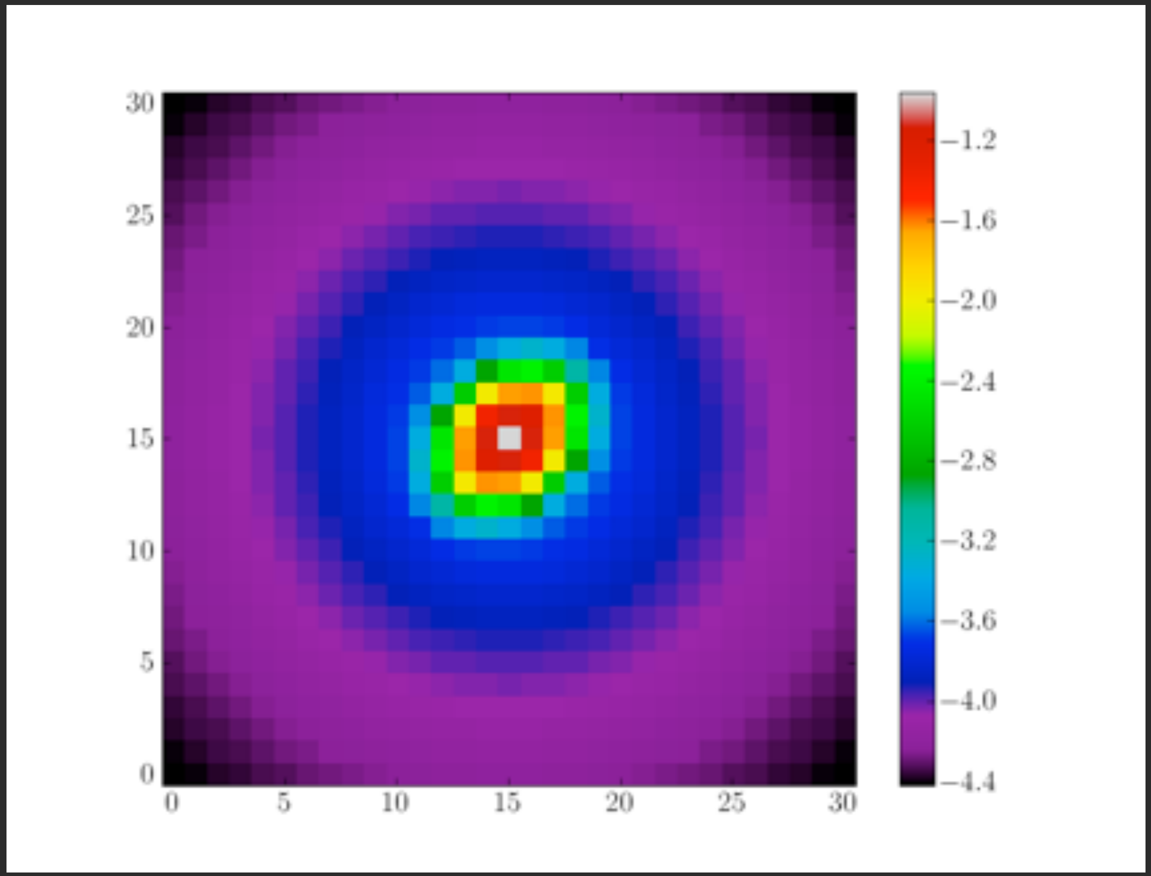
- Wavelength solution
- Spectral trace solution
- Cross-sectional profile
- Relative pixel response
- Line-spread function
- Relative fiber response
- Flux calibration
- Camera aberrations

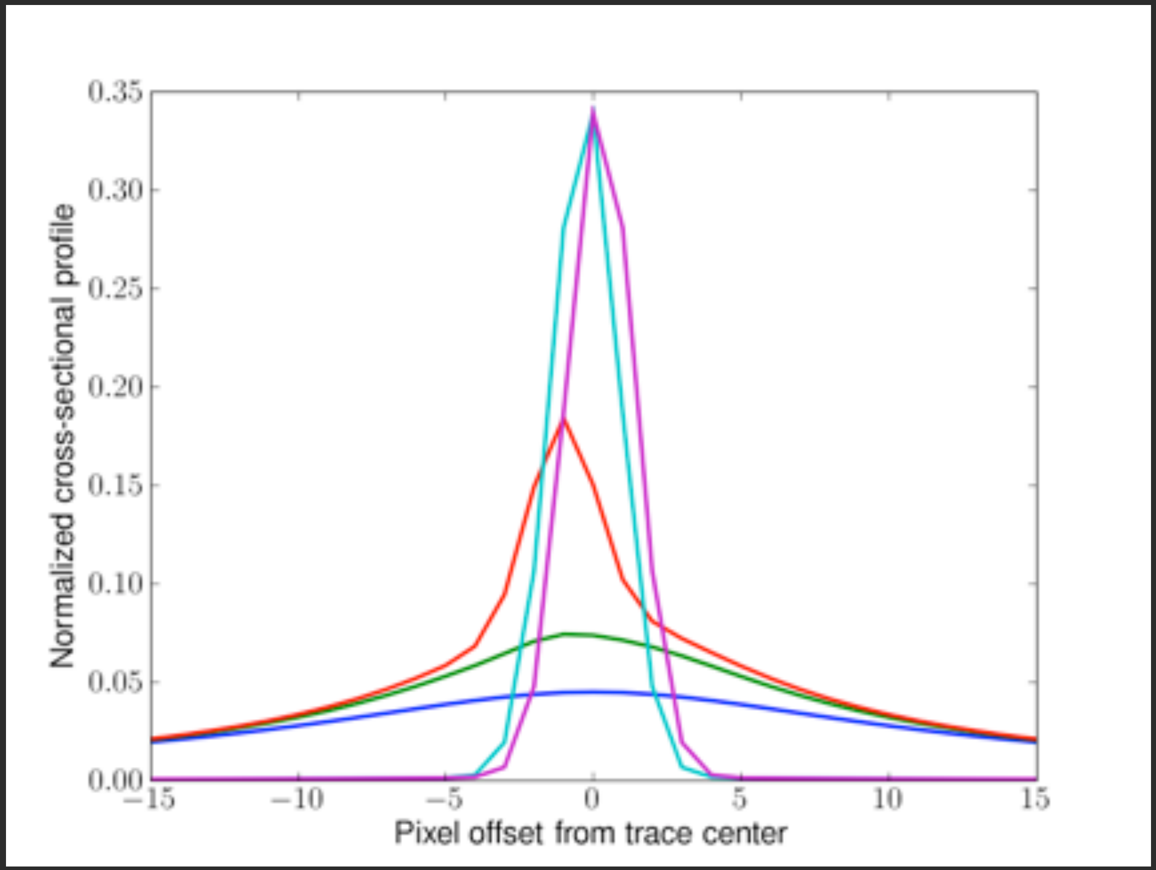
Extraction as image modeling



Model fiber PSF for SDSS1 @ 8500Å

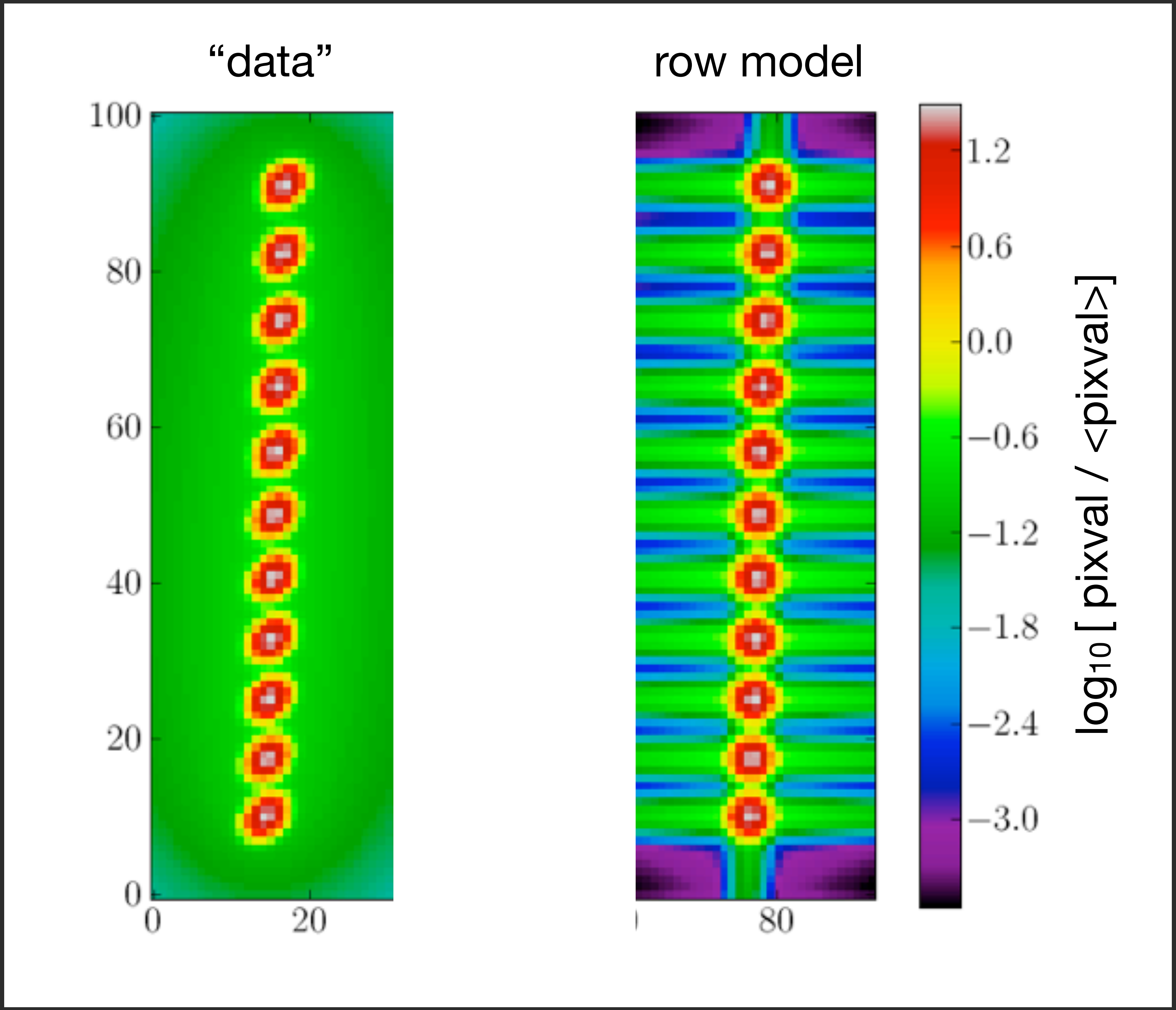
How do you extract an emission line?



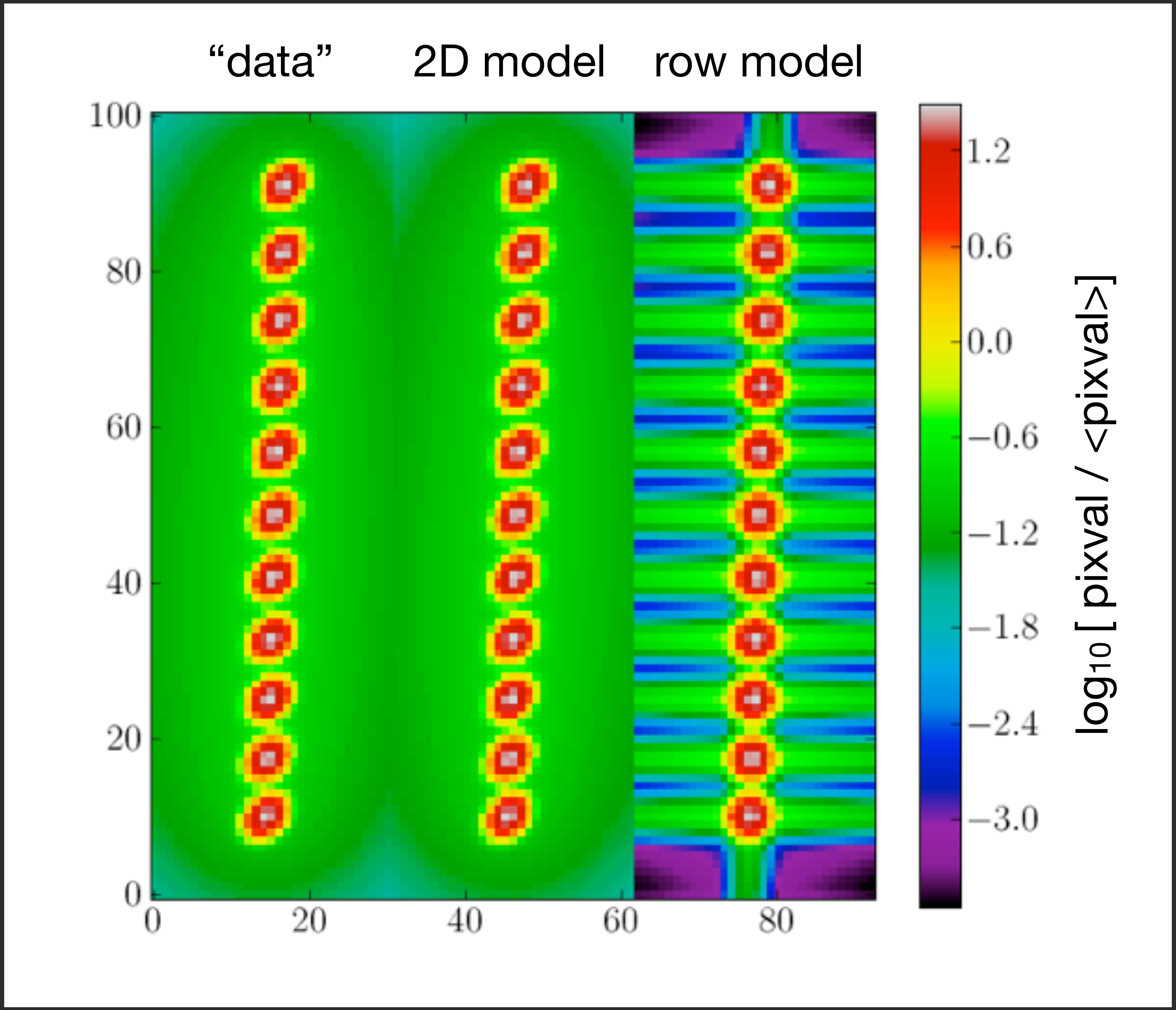


Classic optimal extraction can only be correct when the spectrograph PSF is a *separable* function of x and y.

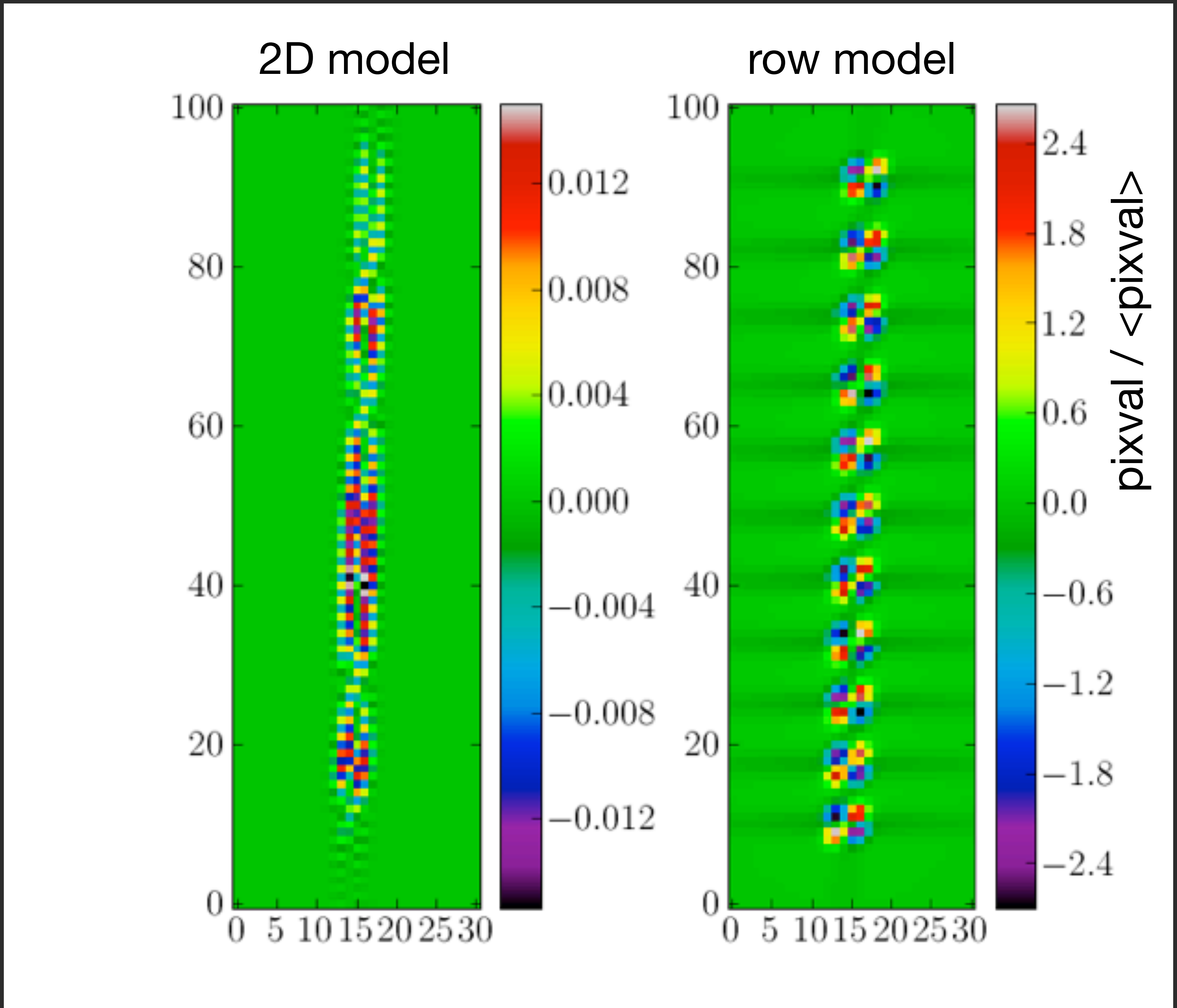
Extraction as image modeling



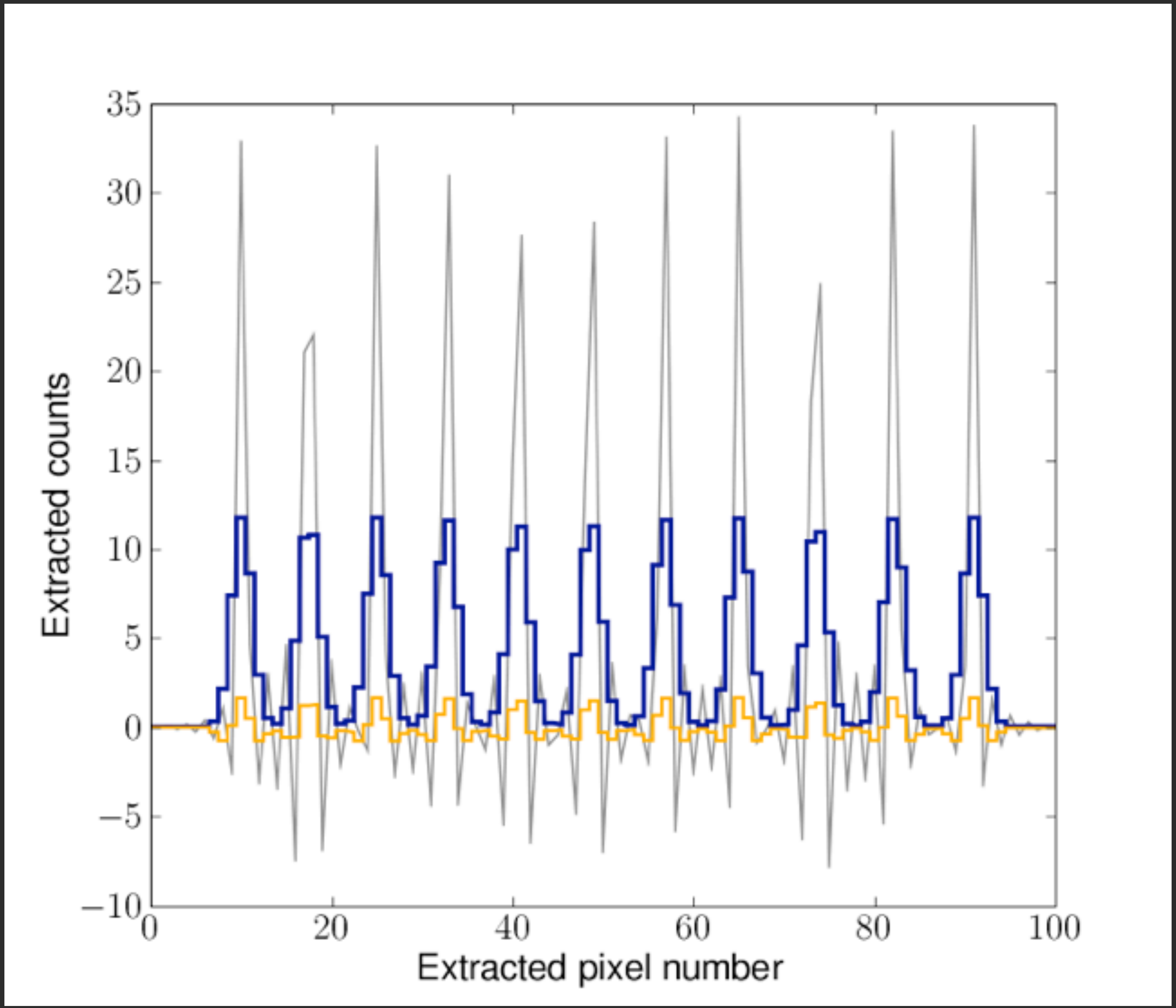
Extraction as image modeling



2D extraction model residuals



Deconvolution and reconvolution



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- n_j: Pixel noise vector
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Put resolution back into spectrum

The formal (deconvolved) solution:

$$f = (A^T N^{-1} A)^{-1} A^T N^{-1} p$$

Inverse covariance matrix of deconvolved spectrum:

$$C^{-1} = A^T N^{-1} A$$

Take unique non-negative square root of this matrix:

$$C^{-1} = QQ$$

Normalize along rows & factor out a diagonal matrix:

$$C^{-1} = R^T \underline{C}^{-1} R$$

By consequence:

$$\underline{C} = R\, C\, R^T$$

The reconvolved spectrum: what *would have been observed* by a 1D spectrograph with same resolution:

$$\underline{f} = R\, f$$

Put resolution back into spectrum

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Note analogy

By consequence:

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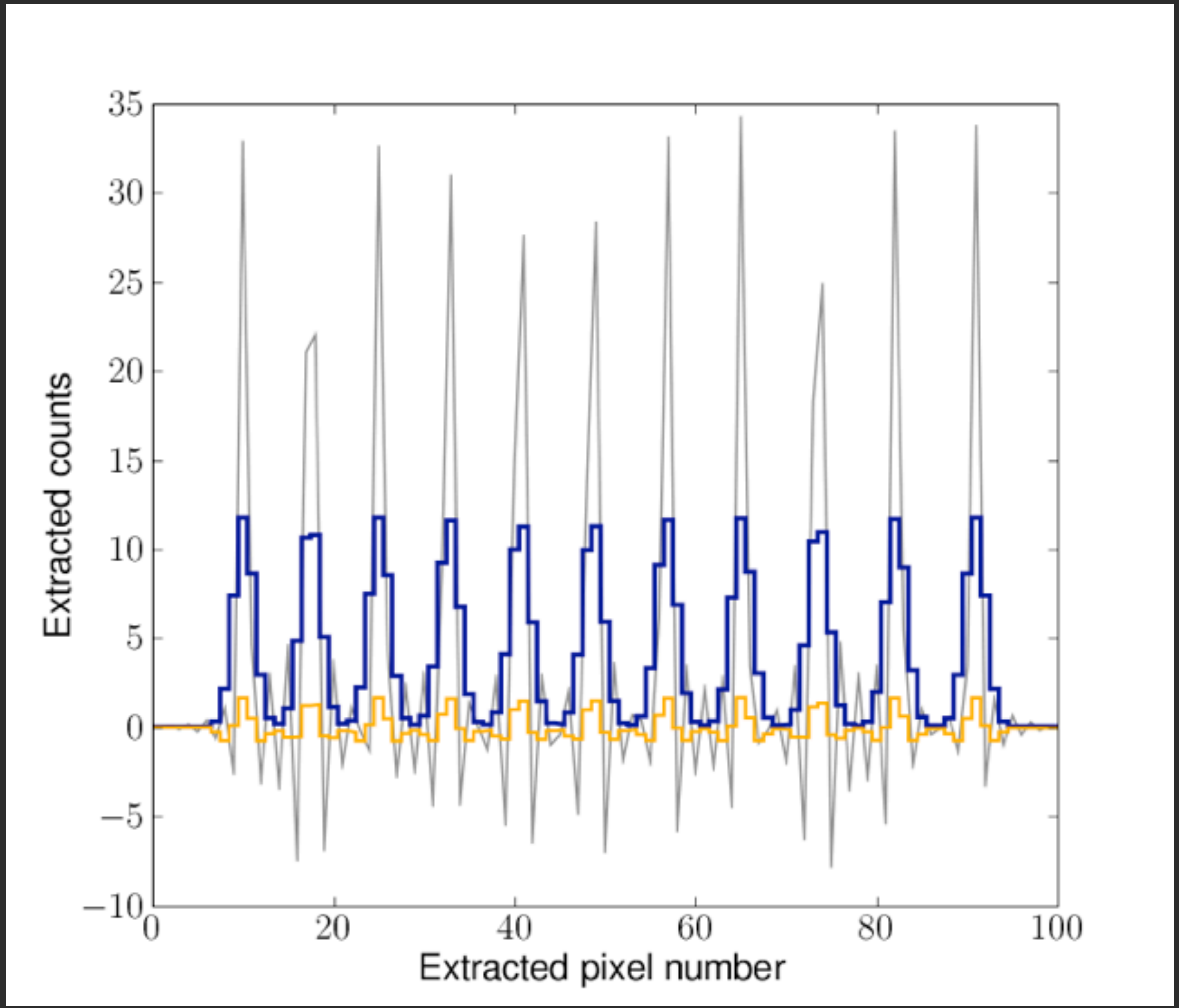
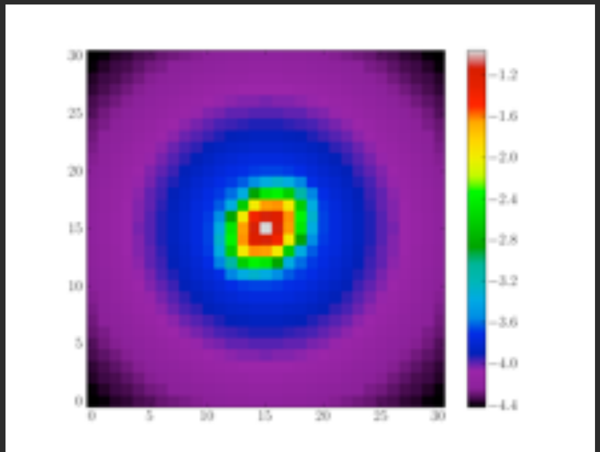
The reconvolved spectrum: what *would have been observed* by a 1D spectrograph with same resolution:

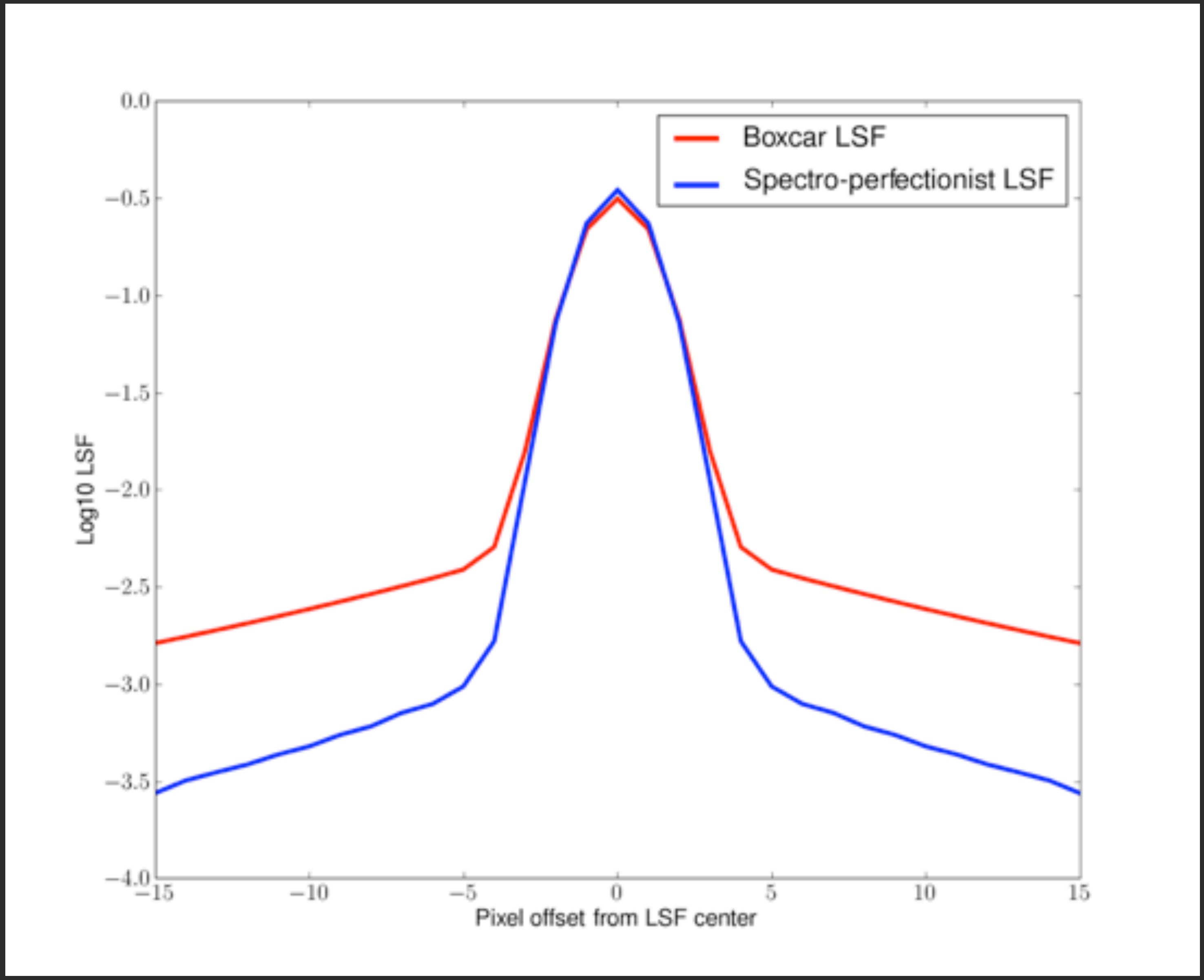
Uncorrelated errors → $\boxed{\underline{f}} = \boxed{R}\, f$ Band diagonal

Deconvolution and reconvolution



Comparative resolution w. r. t. boxcar





To make things interesting, add:

- Noise,
- Variable fiber PSF,
- Multiple frames with flexure/dither,
- “Sky”,
- Fiber-to-fiber crosstalk

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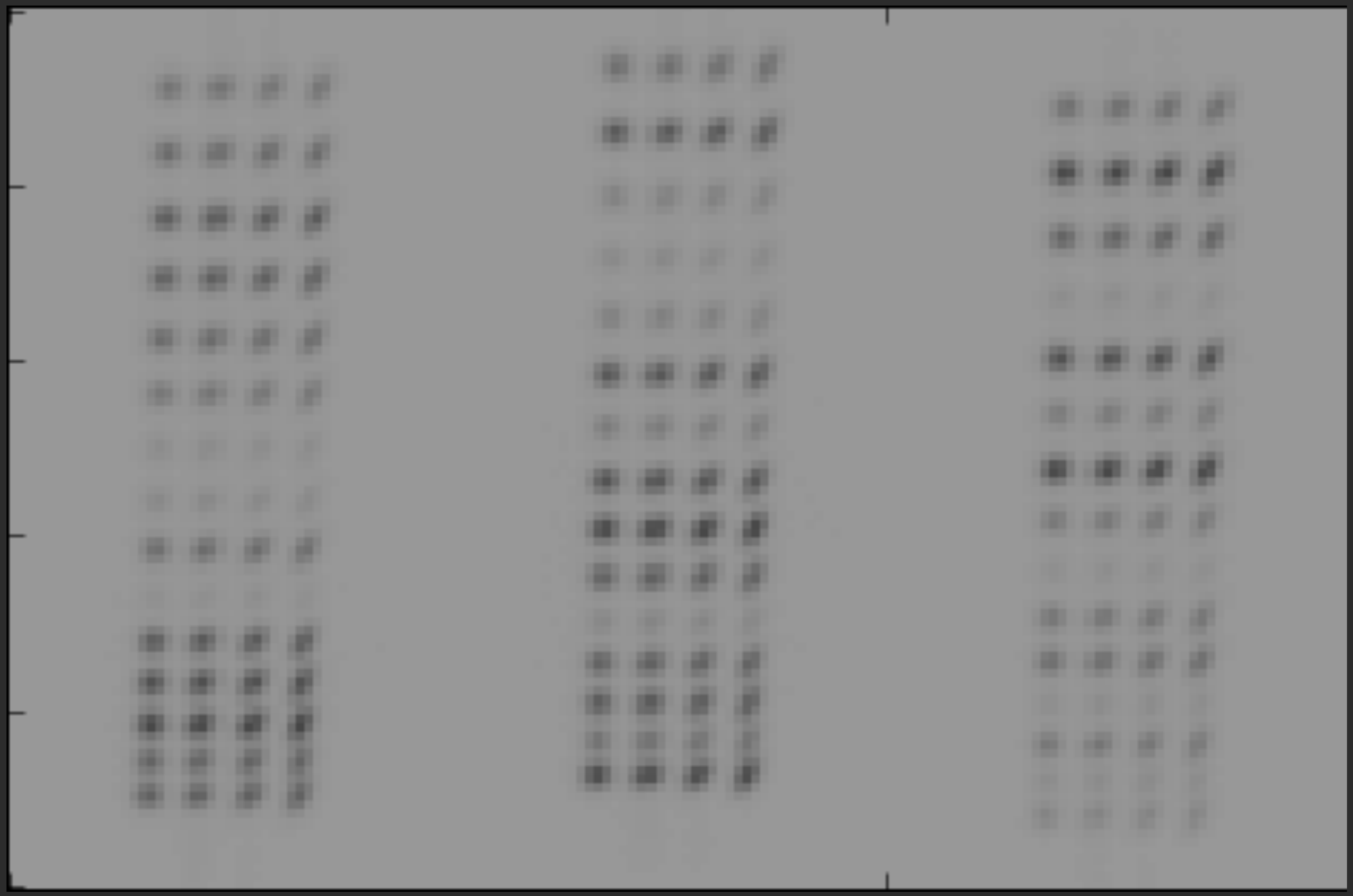
Can do extraction, coaddition, and sky subtraction in one shot!

Multi-frame, multi-fiber simulated data



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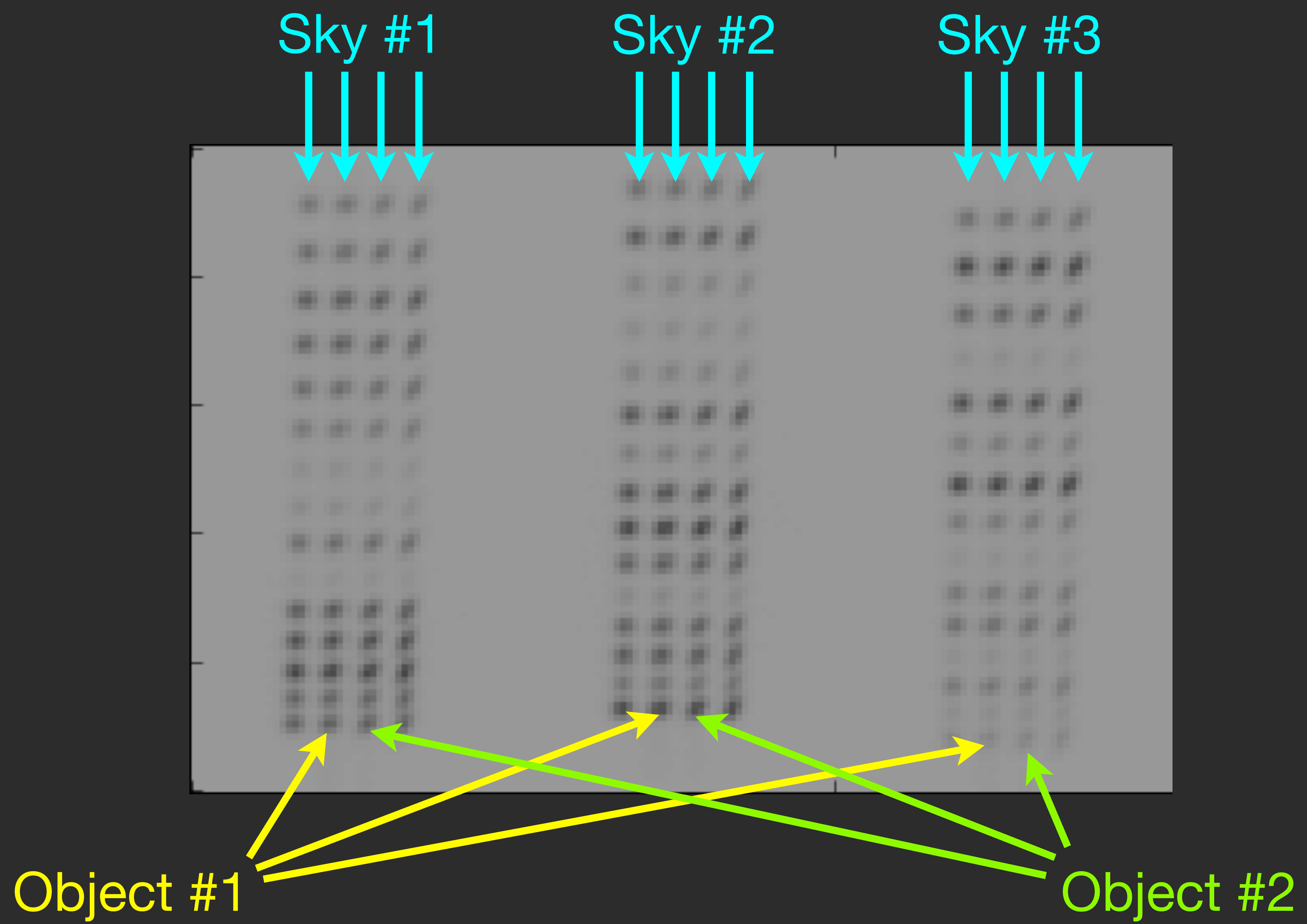

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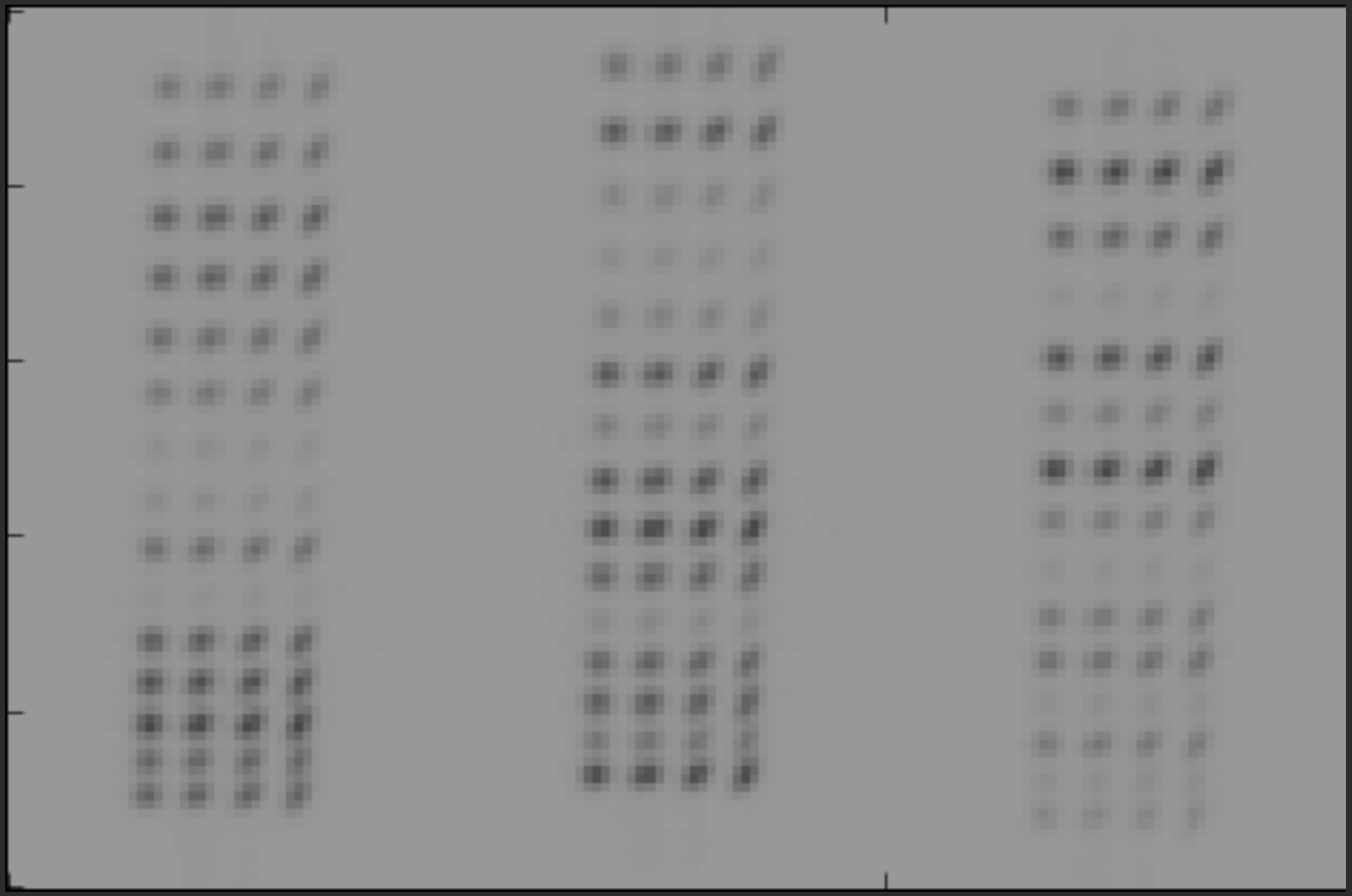
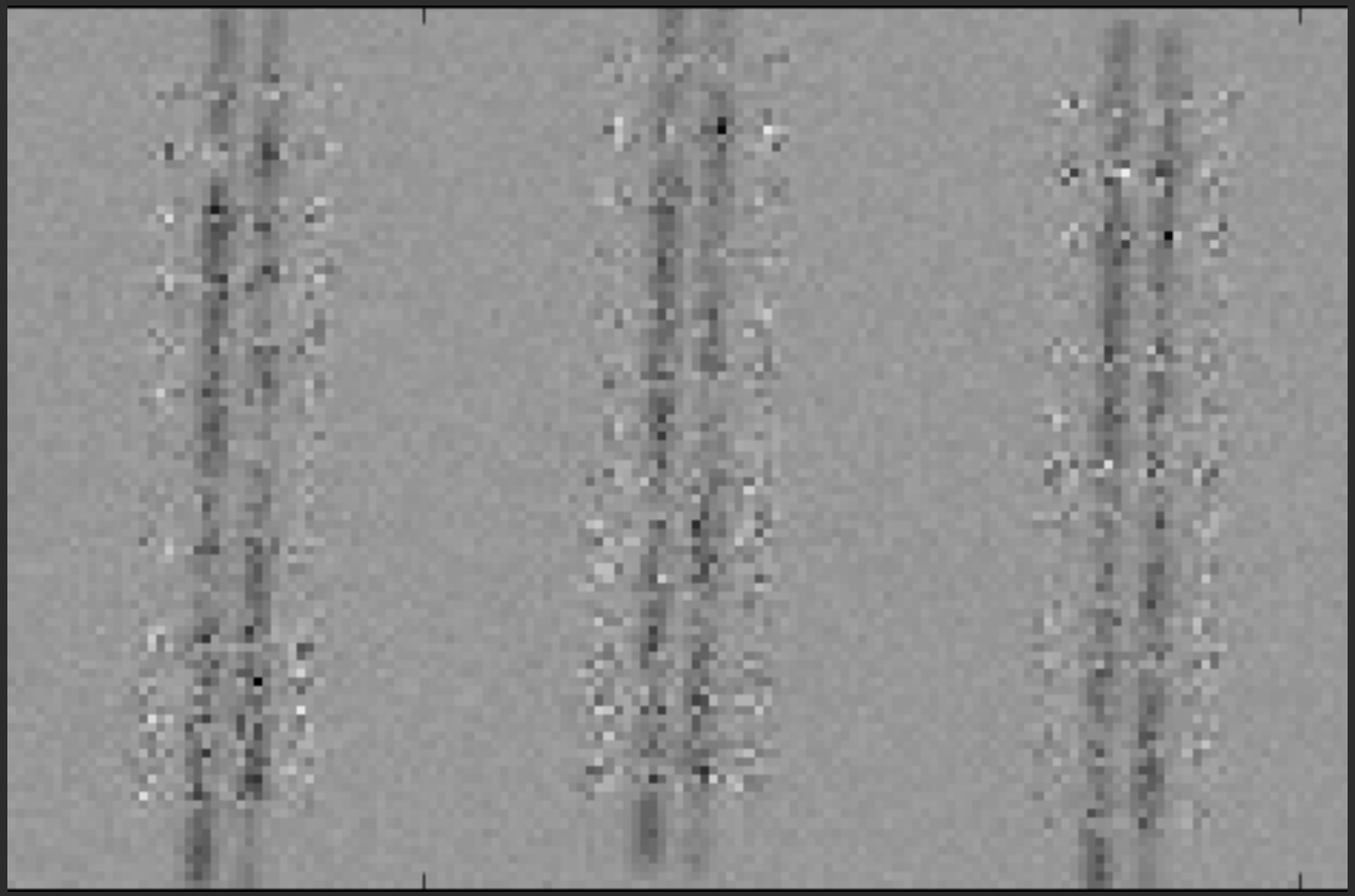


Objflux = Skyflux / 20

ObjSNR ≈ 5 (per extracted sample, sky-noise limited)

Sky model decomposed & removed

Sky spectrum is modeled "upstream" from optical heterogeneities between fibers



(Grayscale stretch X 40 relative to previous)

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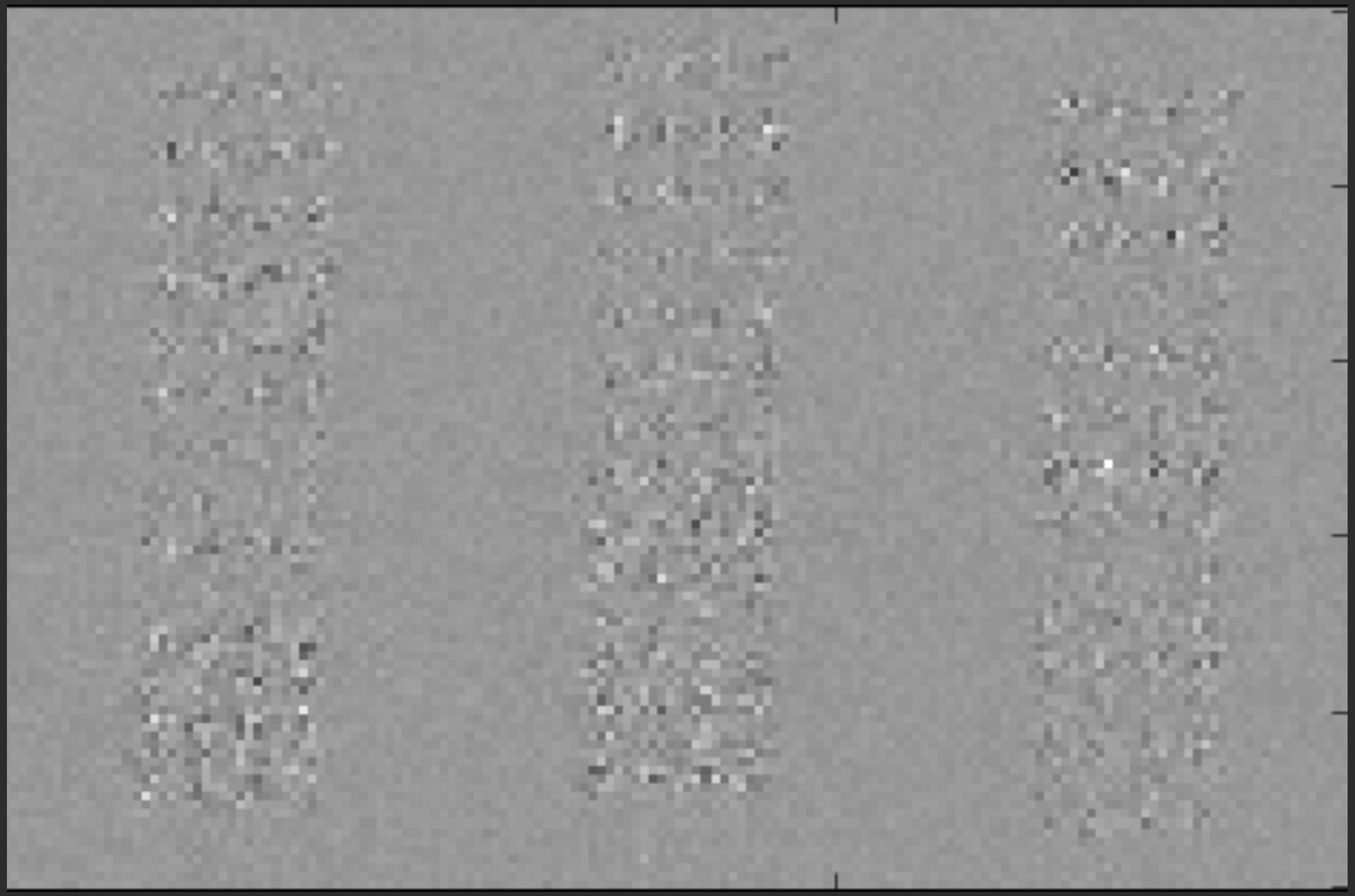
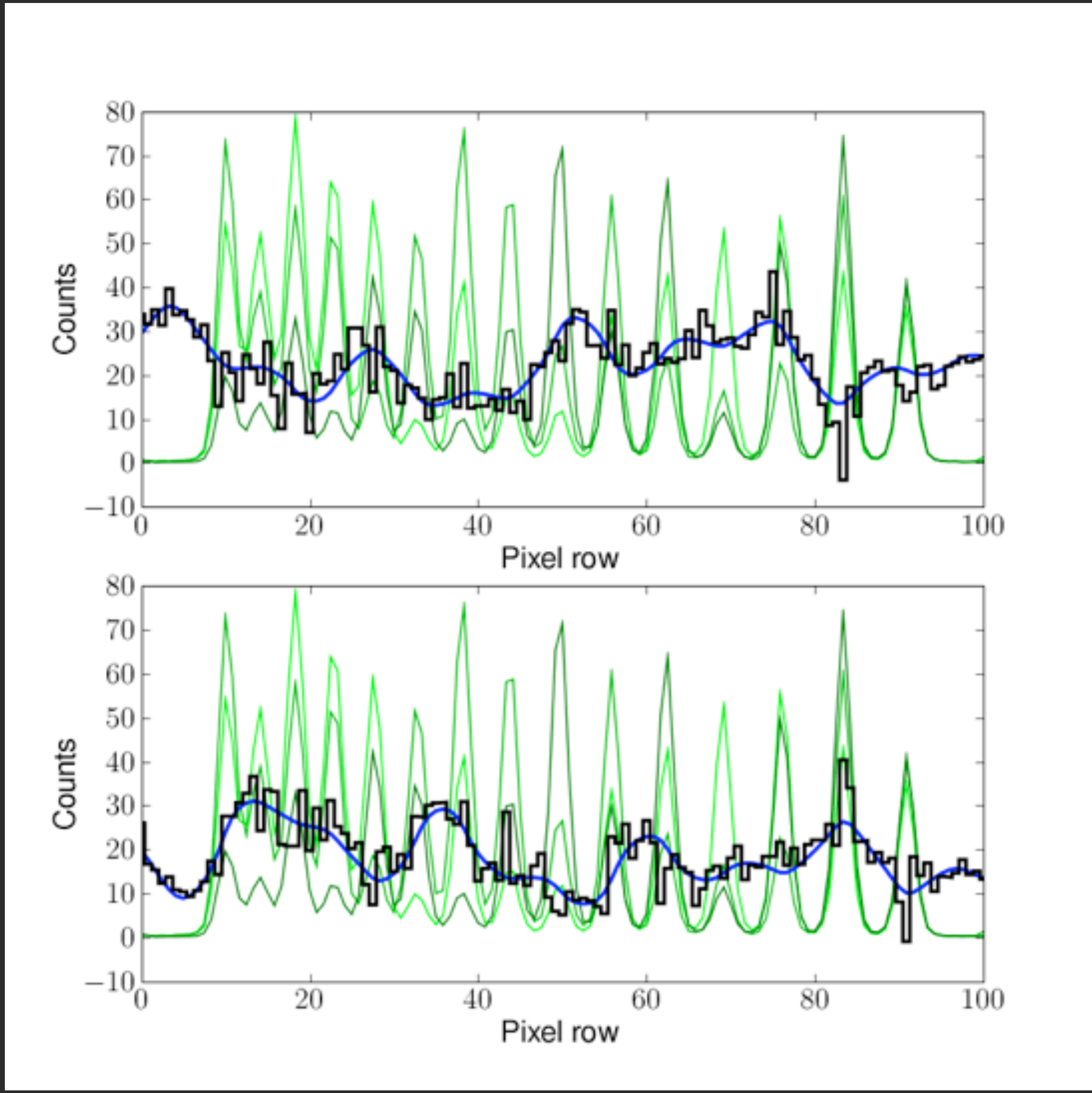
All models removed



Consistent with pure noise

Extracted objects + skies

Sky scaled down by a factor of 20 in plot



RMS error-scaled residuals of unity

Spectro-perfectionism scorecard

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The biggest headache (for me, at least):

Fiber-to-fiber cross-talk couples all spectra.

For each BOSS spectrograph-plate, we have:

500 spectra × 6000 sampling points × 4 frames

⇒ ***12 Million coupled equations to solve!***

Fortunately the matrix is sparse...

(Sampling also swept under rug here, but see paper...)

The challenge to calibration and design

Current calibration facilities may not permit a sufficiently accurate determination of A_{jk}

=> New calibration regimes and equipment (high-wattage monochrometer or tunable laser)

(c.f. Stubbs & Tonry 2006)

Ultimately calls for a full integration of data analysis software with instrumental design software

=> Optimize *scientific* metrics in hardware

=> Tune instrument directly from data

=> “Use what you know” during analysis

The sociological challenge

Site selection:
    Multi-year testing, remote locations, etc.
Telescope:
    As large, reflective, and well-focused as possible
Instrument:
    Expensive design, coatings, high-QE CCDs
Data calibration and extraction:
    Somebody will do something at some point...

What's wrong with this picture?

Summary

- Current extraction algorithms are inaccurate at a level that significantly degrades faint-object fiber spectra
- This problem can be solved with correct 2D modeling
- Resolution is a preservable native attribute of raw data
- Extracted covariance can be made diagonal
- Extraction, coaddition, and sky subtraction in one shot
- chi^2 against spectra <=> chi^2 against raw data
- Immediate application for SDSS-III BOSS
- Very accurate calibration: difficult but important
- Computational challenge is significant
- Check it out: Bolton & Schlegel 2010, PASP, 122, 248

Thank You!

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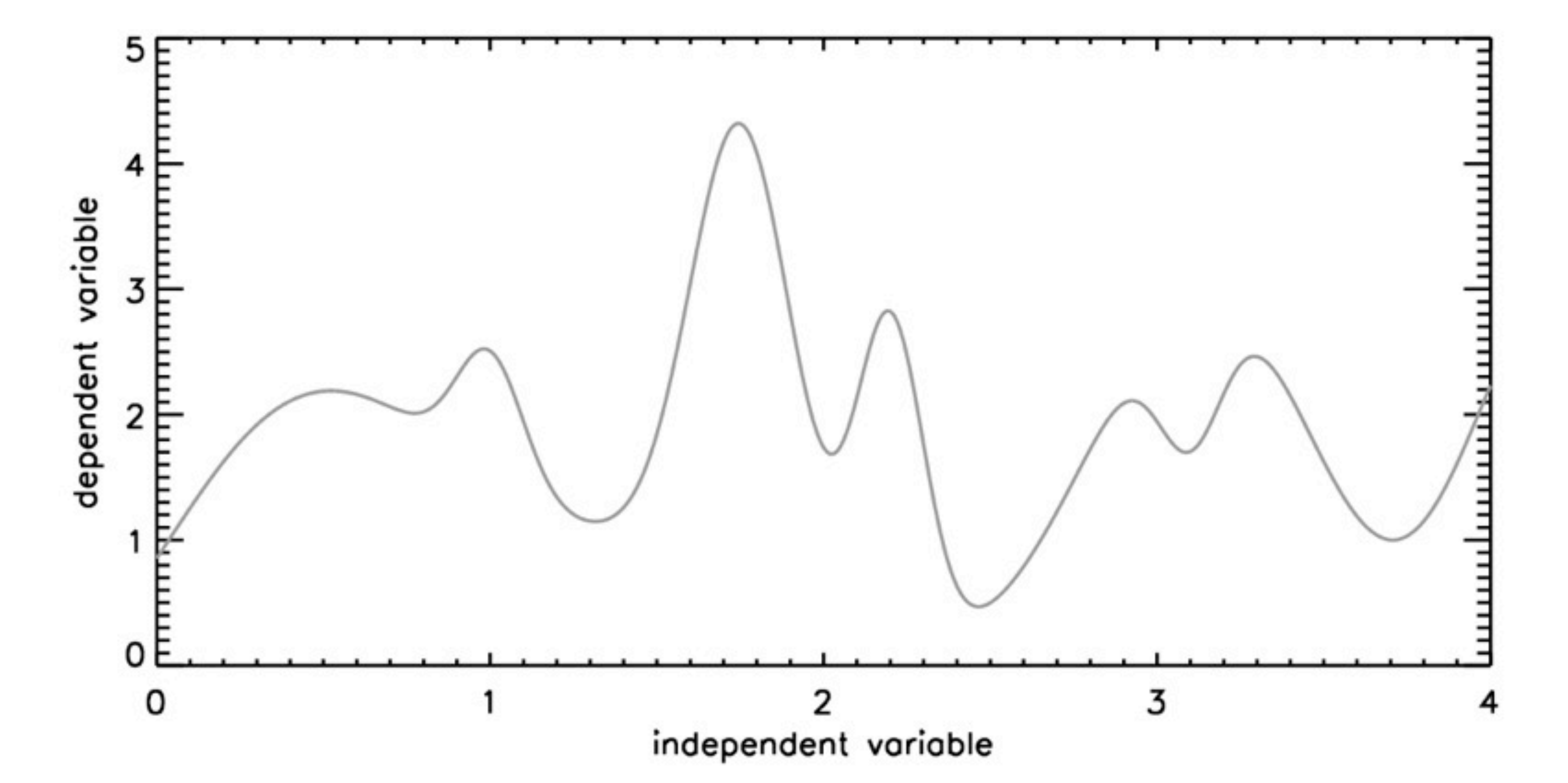
B-splines



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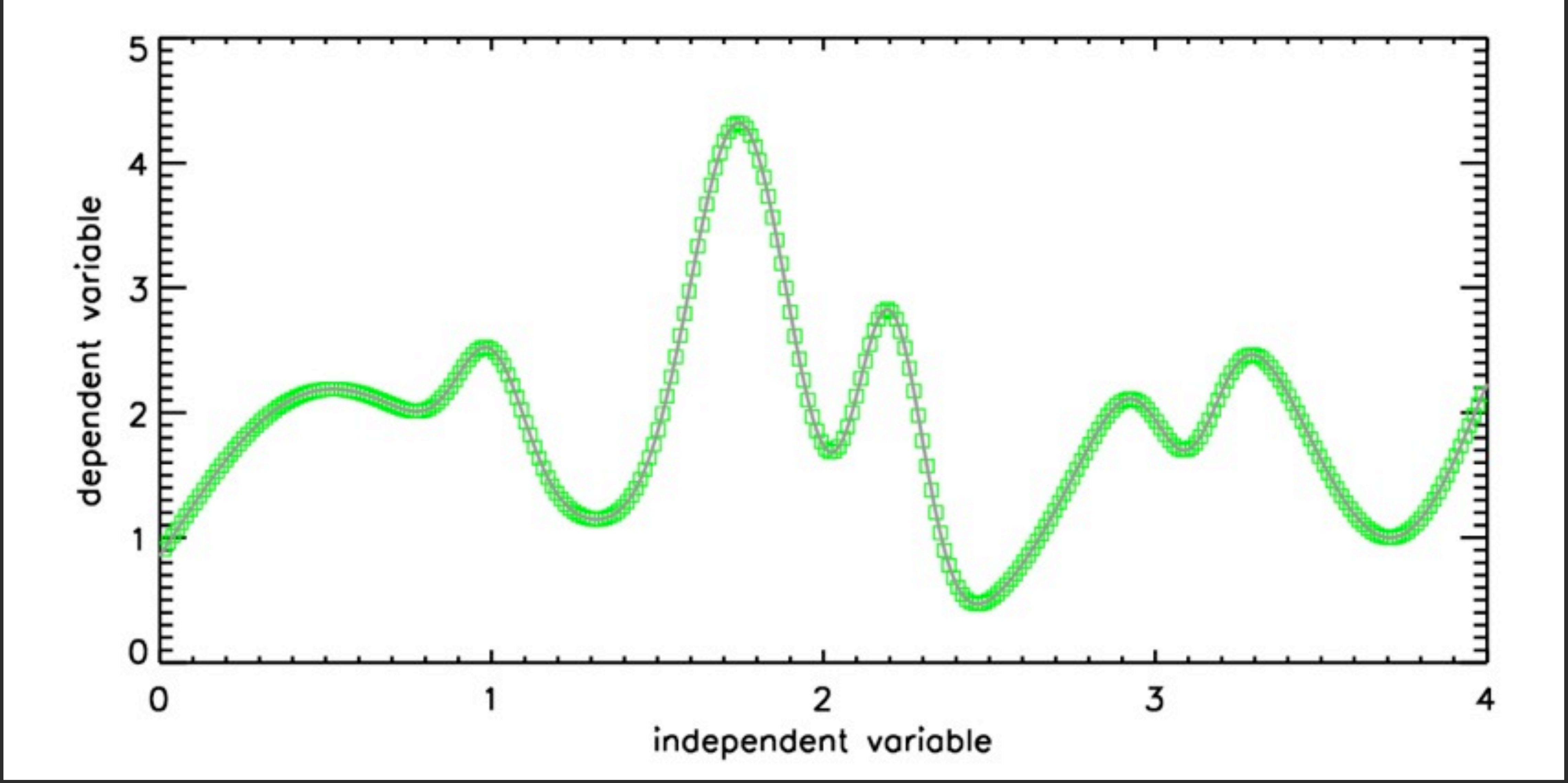

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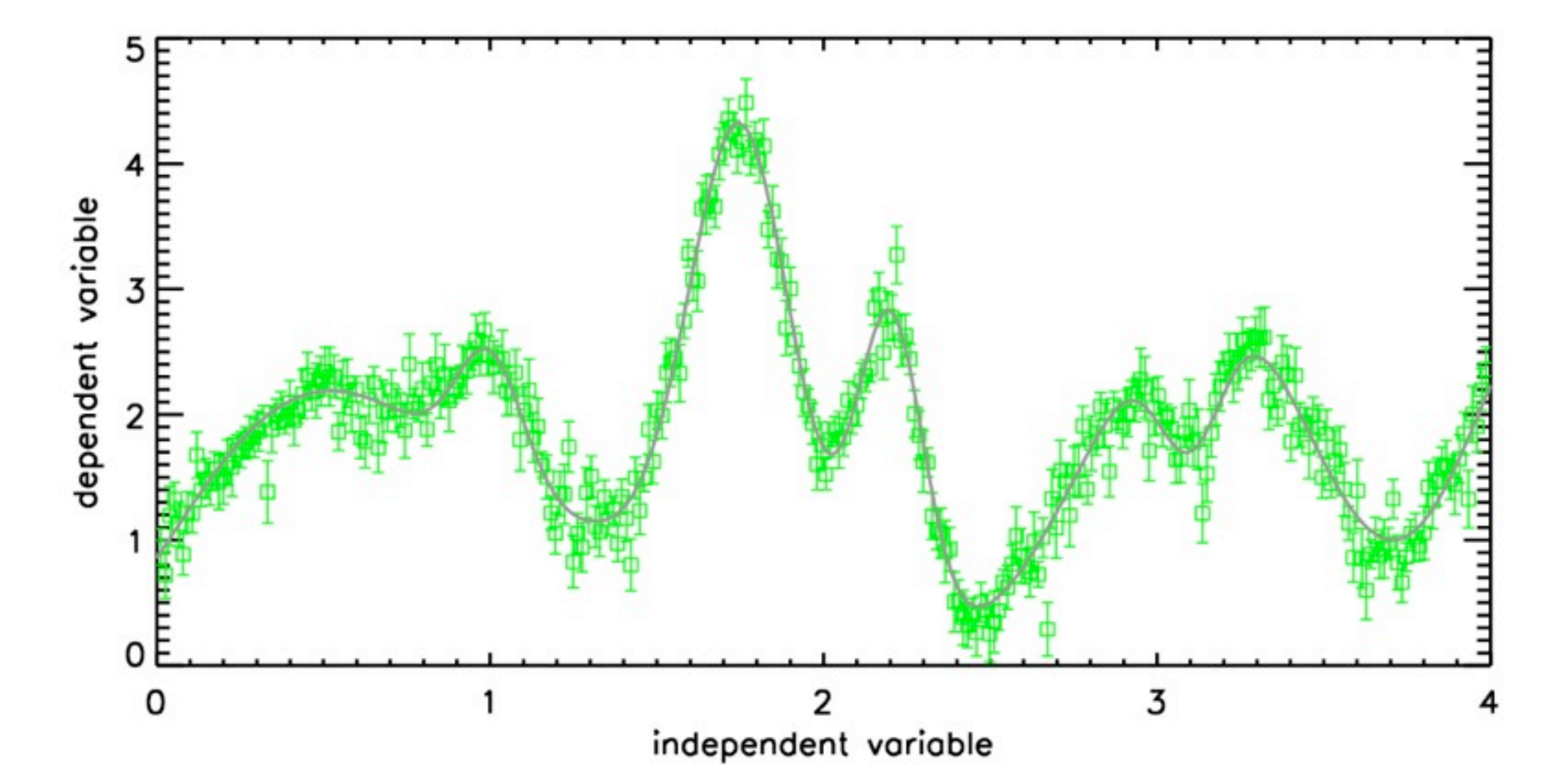

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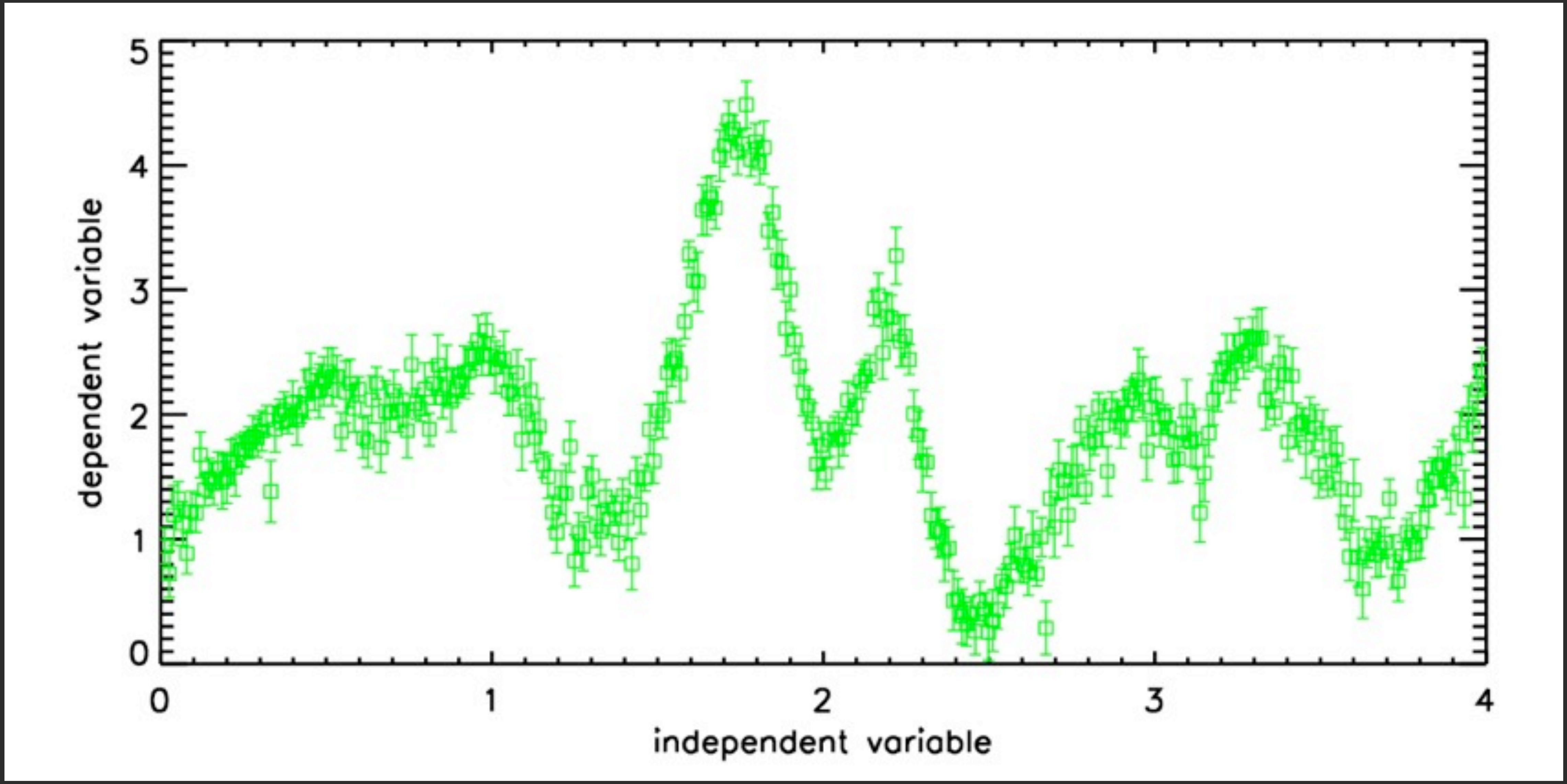

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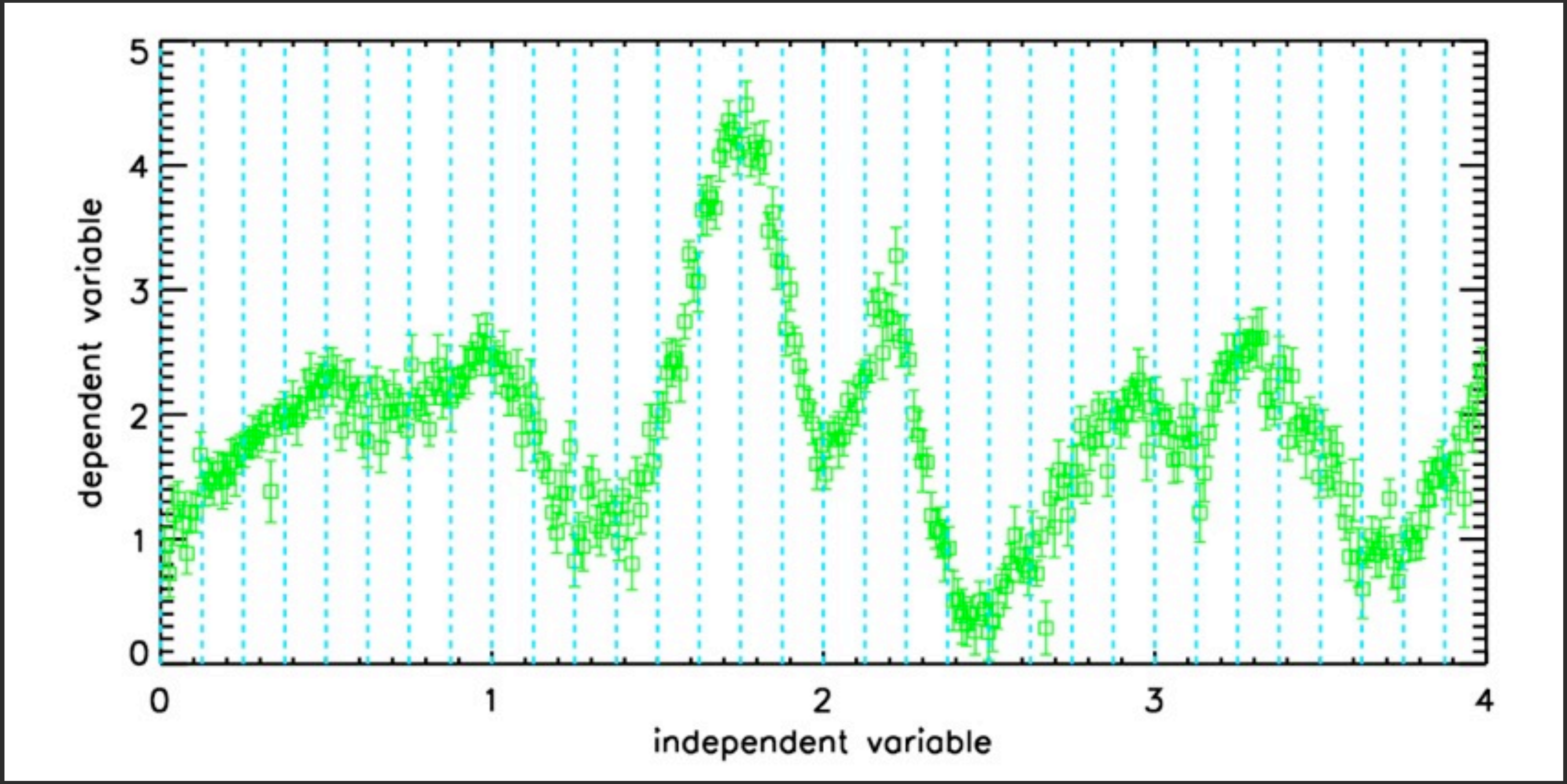

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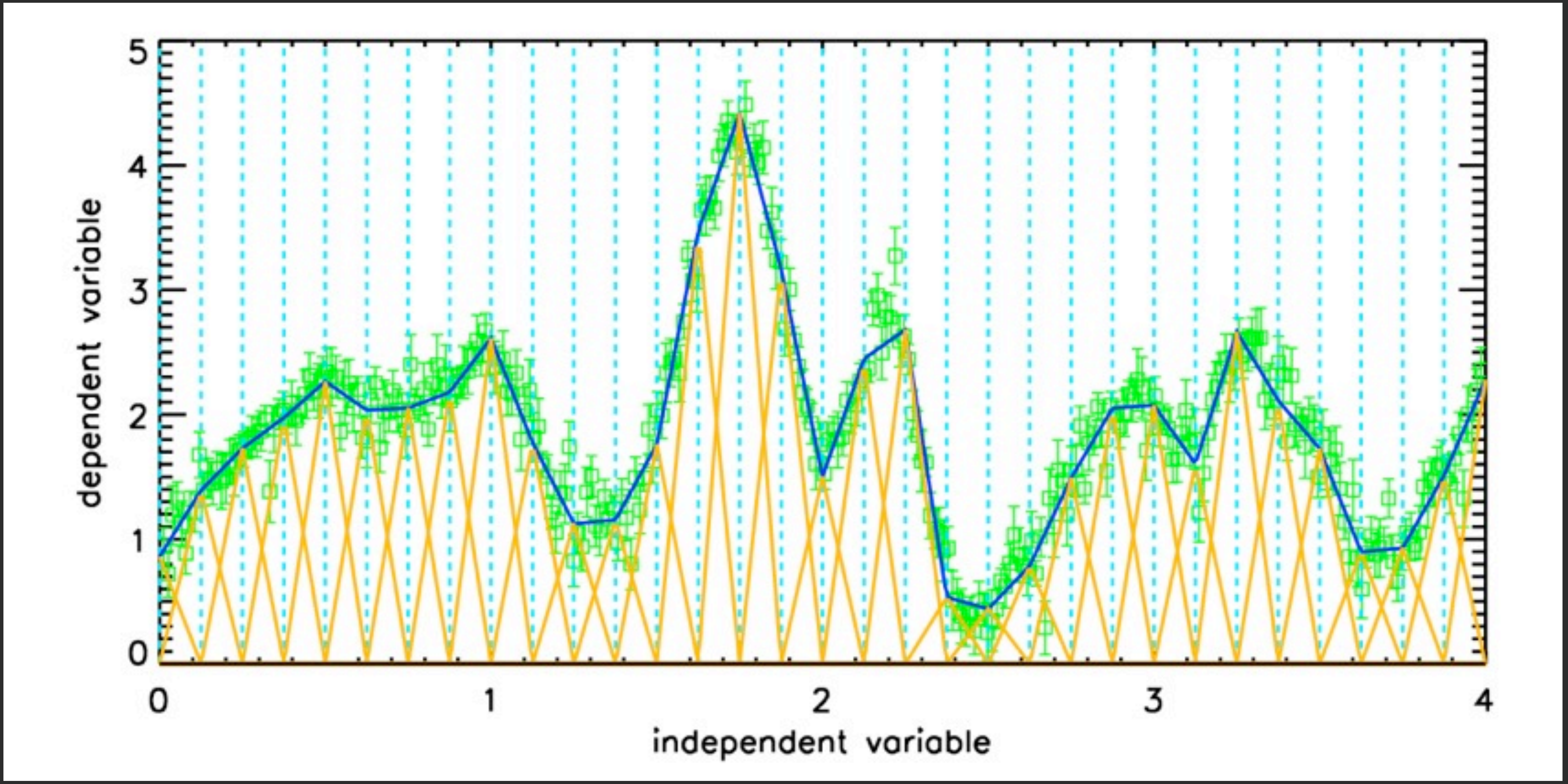

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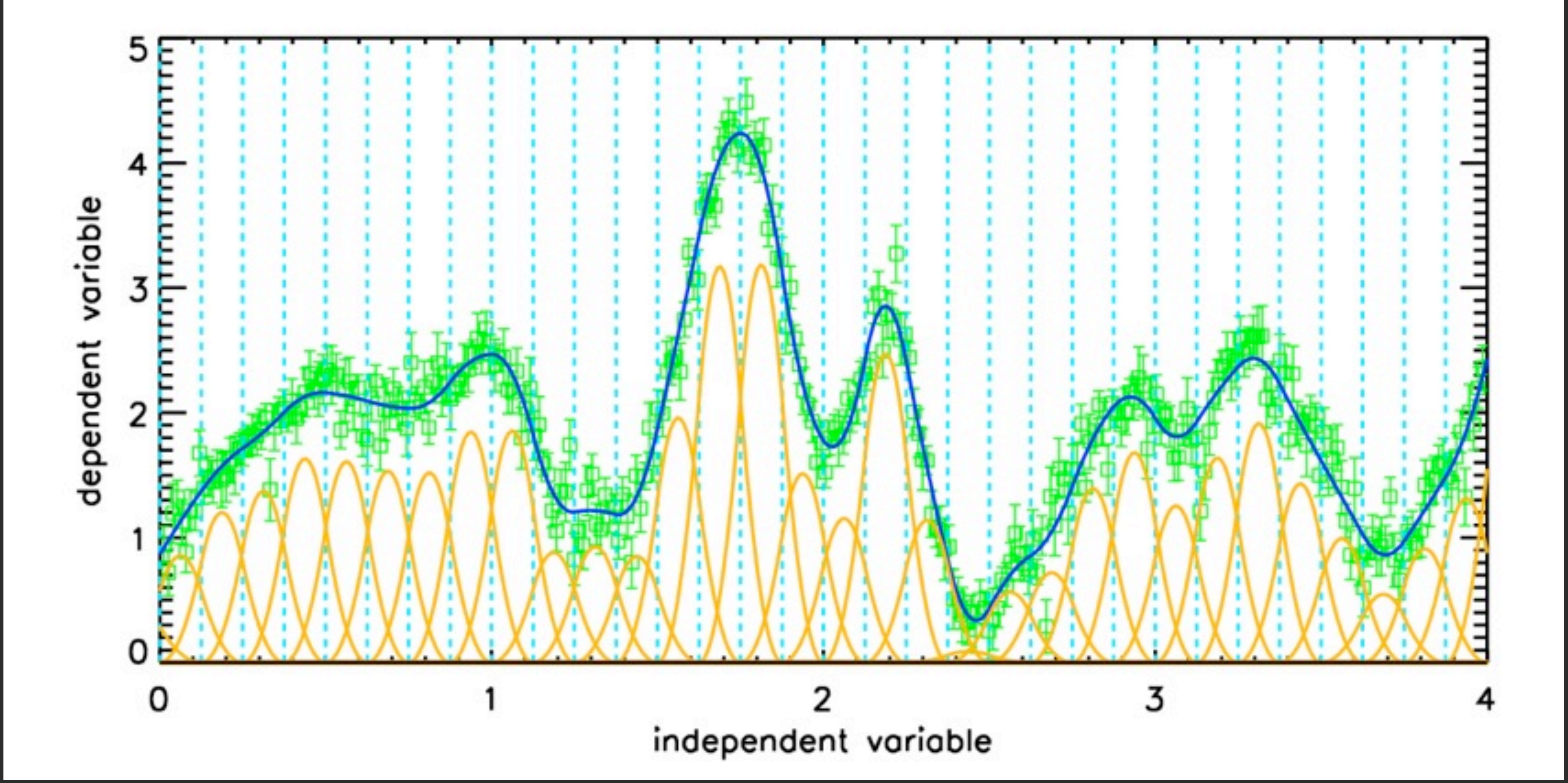

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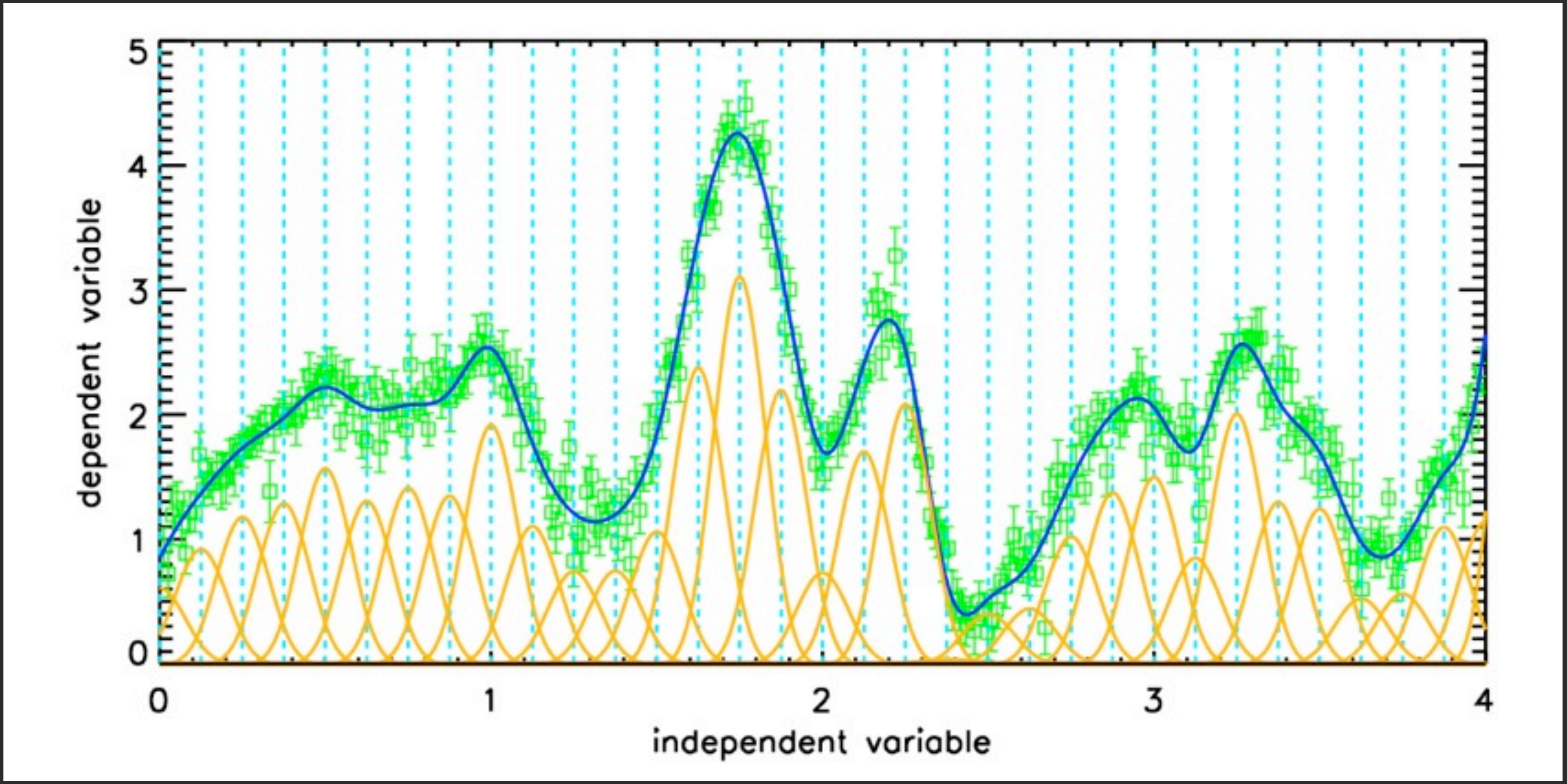

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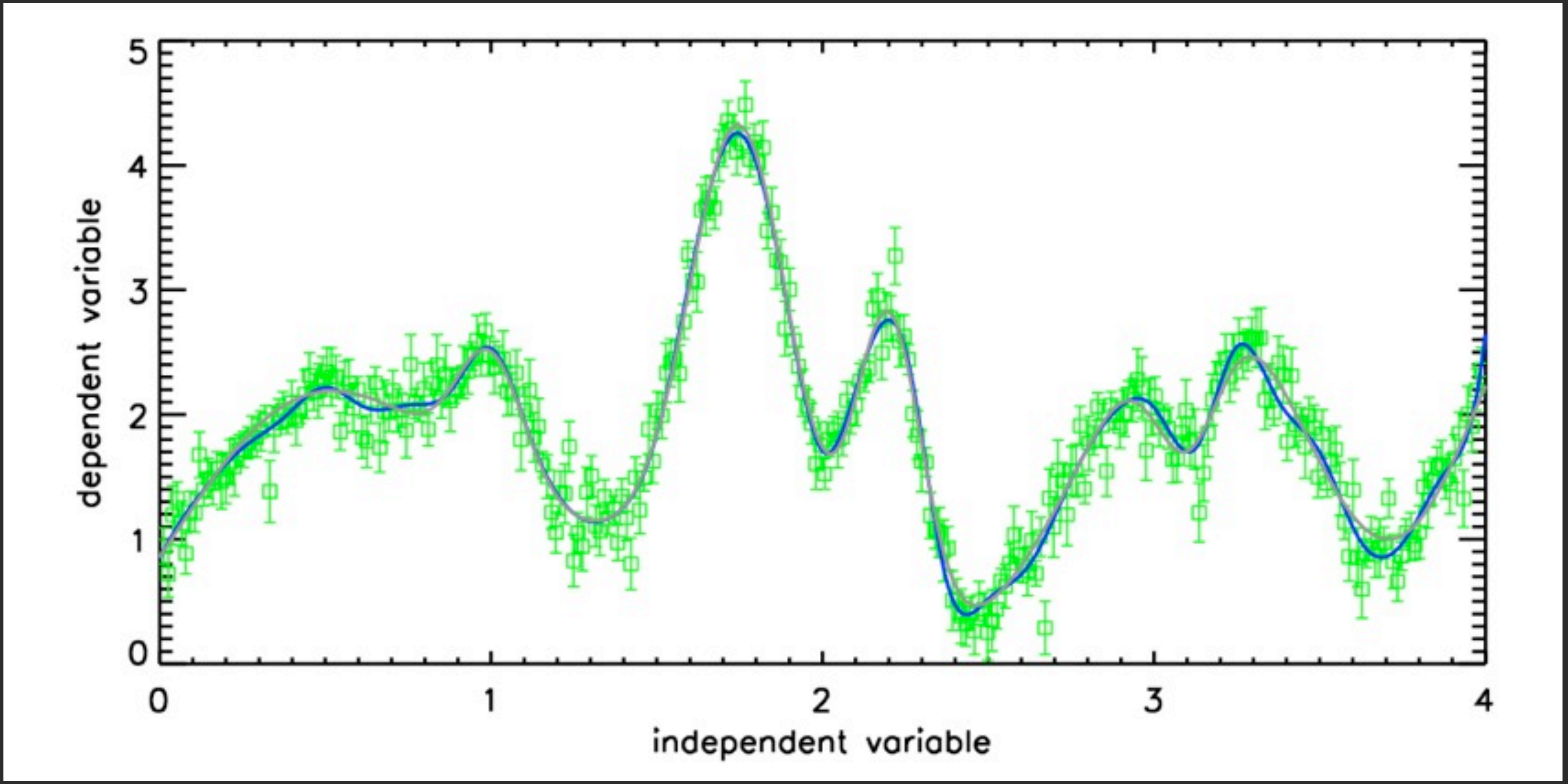

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